Ibn al-Hā'im's Trepidation Model

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[1] This paper was written as part of a research programme, entitled *Astronomical Theory and Tables in al-Andalus and al-Maghrib between the twelfth and fourteenth centuries,* funded by the "Dirección General de Investigación Científica y Técnica" of the Spanish "Ministerio de Educación y Ciencia".

I Introduction

1. Generalities.

The aim of trepidation models was to account for two phenomena that puzzled medieval astronomers: the secular decrease of the obliquity of the ecliptic, and the apparent changes in the velocity of precession over the years[2].

As far as we know, the first theory that attempts to explain the phenomena is found in the *Small Commentary to the Handy Tables* by Theon of Alexandria (4th c.)[3]. Theon's simple original theory, which later on developed into much more complicated models, was based on the belief in a backward and forward motion of the equinoctial points along an arc of 8° at a rate of 1°/80 years, due to which the value of the precession of the equinoxes was sometimes negative and sometimes positive. It seems that the origins of these 8° and of the theory of a linear zig-zag function are to be found in ancient Babylon. Theon attributes the theory to the "ancient astrologers", a term that was translated in Arabic works as "Ahl al-ṭilasmāt". Theon's words were often reproduced, in various ways, by the Arabic sources in which the theory was developed.

According to J. Ragep[4], al-Battānī played a central role in its development. Following earlier Islamic authors of the 9th century, he made a reinterpretation of the theory described by Theon and provided the framework within which later models would develop.

We can identify two different approaches: the Eastern approach due to Ibrāhīm b. Sinān b. Thābit b. Qurra (*Kitāb Ḥarakāt al-Shams*) and Abū Jaᶜfar al-Khāzin (*al-Zīj al-Ṣafā'iḥ*), both from the 10th century, and the Western one, mainly represented by the *Liber de Motu Octave Spere*, whose date of composition is not known; the trepidation tables found in the

[2] A full account of the most recent bibliography on this subject is to be found in J. Samsó [1992:219-225] and [1994:VIII,1-31], J. Ragep [1996:267-298], R. Mercier [1996:299-347], and M. Comes [1996:349-364].

[3] See A. Tihon [1978:236-237/319].

[4] See J. Ragep [1996:271-295].

Toledan Tables[5]; and the work of Azarquiel[6] and his followers, from 11th century onwards[7]. The main difference is that while in the *Kitāb Ḥarakāt al-shams* and in *al-zīj al-Ṣafā'iḥ* we find just purely geometrical demonstrations without parameters, in the Western authors there is a complete set of parameters as well as tables to compute the position of the Head of Aries and the obliquity of the ecliptic for any desired moment.

To this, it should be added that in the trepidation theories of the Ancients, the solstitial points are the fixed reference. The same occurs in the aforementioned works of Sinān b. Thābit[8] and al-Khāzin[9]. In Zarqāllian and Andalusian models, the reference system is defined by the equinoctial points.

Andalusian models were probably reelaborations of these earlier Eastern models with the attribution of parameters that accounted for the differences found by the observations. In fact there is a paragraph in Azarquiel's introduction to his *Book on the Fixed Stars* where there seems to be an indirect reference to Thābit. Azarquiel[10] states that the previous authors did not pursue the subject of the motion of the fixed stars further because reliable observations of earlier astronomers were not available to them.

According to J. Ragep, presumably "one of Thābit's reasons for writing Isḥāq[11] was to ascertain if he knew of any observation of the sun between

[5] See G.J. Toomer [1968:118-122].

[6] Following J.M. Millás [1943-1950], I use the Spanish form *Azarquiel*, deriving from the Arabic *walad al-Zarqiyāl*, documented by Ṣāᶜid al-Andalusī [2:138-139/3:180-181].

[7] See chapter 1 of the following commentary.

[8] See Saᶜidān [1983].

[9] Ms. Asrinagar, 314. We owe a photocopy of the manuscript to the kindness of Professors Ansari and King. This *Zīj al-Ṣafā'iḥ* is mentioned by al-Bīrūnī in his *Kitāb al-Athār al-Bāqiya* [1:326] as containing a good description of an accession and recession model.

[10] J.M. Millás [1943-1950:278].

[11] He is referring to a scientific epistle that Thābit wrote to Isḥāq b. Ḥunayn, the well-known physician and translator from Greek into Arabic, kept in Ibn Yūnus's *al-Zīj al-Ḥākimī*.

Ptolemy and their own time" in order to test whether trepidation could account for "the differences between the *Mumtaḥan* results and those of Ptolemy to a generalized phenomenon affecting all the stars"[12]. Furthermore, Thābit ends the letter saying that he has not communicated his studies on that respect to anyone "even if many have asked me for it, particularly because they are not found on something precise, but I establish now the point to which the question has evolved, from Ptolemy to this time epoch which is ours"[13].

Azarquiel also criticizes the efforts of previous Andalusian astronomers, especially Ibn al-Samḥ (d. 1035)[14]. Indeed he seems to feel a particular animosity towards Ibn al-Samḥ, because similar negative judgements are also found in an indirect reference in his book on the use of the equatorium[15].

However, the fact that we find here a statement affirming that Ibn al-Samḥ had reached an understanding of the motion of the fixed stars, albeit incomplete, raises the possibility that he was the author of the model of the *Liber de Motu*, on which Azarquiel based his own models. Moreover, in Ṣāᶜid's *Ṭabaqāt*[16], Ibn al-Samḥ is described as a specialist in mathematics and geometry[17], as well as in the motion of the stars and the configuration of the celestial spheres.

Another possibility is Ṣāᶜid himself for in his *Ṭabaqāt*[18] he states that after reading Ibn al-Ādamī's treatise on trepidation, he was able to understand trepidation motion and to expound it in his book on the motion

12 J. Ragep [1996:282-283]. See also R. Morelon [1994:131-132].

13 R. Morelon [1994:132].

14 J.M. Millás [1943-1950:278].

15 See M. Comes [1990:225].

16 Abū l'Qāsim Ṣāᶜid al-Andalusī's *Ṭabaqāt al-Umam*. See Ṣāᶜid al-Andalusī [3:169-170].

17 Ibn al-Samḥ's skill as mathematician and geometer, especially as far as his study on the sections of the cylinder is concerned, has been confirmed by R.Rashed's outstanding study [1996].

18 Ṣāᶜid al-Andalusī [2:114] and [3:146-47].

of the celestial bodies. This might explain the presence in the *Toledan Tables*, also attributed to Ṣāᶜid and his group, of the same trepidation tables we find in several manuscripts of the *Liber*.

Finally, other possibilities are: Ibn Barghūth, whose observations of the star *Qalb al-Asad* (441H/1049-1050AD) are quoted by Azarquiel[19]; or, better still, al-Istijjī[20], who according to Ibn al-Hā'im wrote a *Risālat al-iqbāl wa-l-idbār* (c. 1050) and seems to have devised his own trepidation model.

There is no doubt, however, that Azarquiel remains one of the most likely candidates as author of this book. Of course, the fact that Ibrāhīm b. Sinān does not attribute any observations or theoretical work on that subject to his grandfather is for me conclusive evidence that the *Liber* was not due to Thābit b. Qurra. Be that as it may, it is beyond question that the *Liber* is intimately linked with the *Toledan Tables*[21].

Some other authors describe a combination of constant precession and variable trepidation, a combination well known in al-Andalus. In fact, the authors of the *Alfonsine Tables* used a procedure in which a constant precession is combined with a trepidation model, placing themselves in the Andalusian tradition, which is probably derived from al-Battānī's reinterpretation of Theon's statement in this respect[22].

J. Ragep, in his notable study of Naṣīr al-Dīn al-Ṭūsī's *Tadhkira*[23], shows that al-Ṭūsī evokes this hybrid model in his book and adds that Shīrāzī and other commentators raised objections. Al-Ṭūsī, as we shall see, presents other features which are basically Andalusī[24].

[19] J.M. Millás [1943-1950:309].

[20] *Qāḍī* Ṣāᶜid al-Andalusī in his *Ṭabaqāt al-Umam* also states that Abū Marwān al-Istijjī had studied the subject of trepidation. See H. Būᶜalwān's edition (Ṣāᶜid al-Andalusī [3:196-7]).

[21] On the authorship of the *Liber de Motu*, see the account in J. Samsó [1992:225/1994a:2-3]; J. Ragep [1993-400-403] and R. Mercier [1996:321-325].

[22] See J. Samsó [1987a:IXI/175-183], and J. Ragep [1996:267-298].

[23] J. Ragep [1993].

[24] See the commentary introduced after II.4.[5].

We also have the writings of some Maghribī authors of the 15th and 17th centuries, which provide information about some astronomical activity in al-Maghrib between the 12th and the 14th centuries. In the light of the disagreements between computation and observation, they consider that trepidation theories are no longer applicable or at least that another motion should be added to accession and recession[25]. Even Copernicus was to propose superimposed motions of this kind in Book III of *De Revolutionibus*[26].

2. Aim of the paper.

The aim of this paper is to study the trepidation models described in *al-Zīj al-Kāmil fī 'l-Taᶜālīm* by Abū Muḥammad ᶜAbd al-Ḥaqq al-Gāfiqī al-Ishbīlī, known as Ibn al-Hā'im[27].

The only copy of this *zīj* is MS. Arab 285 (Marsh 618), kept in the Bodleian Library. It consists of 170 pages and appears to have no tables, only the canons divided into seven books (*maqālāt*), after quite a long introduction. *Al-Zīj al-Kāmil* was composed ca. year 601 *Hijra* (1204-5), probably in al-Maghrib or al-Andalus, and was dedicated to the Almohad caliph Abū ᶜAbd Allāh Muḥammad al-Nāṣir (1199-1213).

In this book, Ibn al-Hā'im seems to describe all he knows of the trepidation and obliquity of the ecliptic models developed in al-Andalus, especially Azarquiel's third model. He provides not only the description of the model, the geometrical demonstrations, and the use of the tables, already found in Azarquiel's *Book on the Fixed Stars*, but also the spherical trigonometrical formulae involved.

[25] See J. Samsó [1998] and M. Comes [1997].

[26] See in the respect W. Hartner [1984:277]. On Copernicus's trepidation and its relationship with the Arabic tradition see Swerdlow and Neugebauer [1984:1:42-43;61, 72-74 and 127-172]

[27] On this author see: E.S. Kennedy [1956:132(n.48)]; J. Samsó [1992:320-325]; M. Abdulrahman [1996a:365-381]; E. Calvo [1998:51-111 and [1997]; and R. Puig [2000:71-72].

3. Structure of the paper.

3.1 Edition of the text.

An edition of the sections of the text dealing with the trepidation of the equinoxes and the obliquity of the ecliptic appears as an appendix at the end of the paper. Each chapter has been divided into paragraphs, numbered in square brackets. The change of folio is noted in parentheses.
As the manuscript is a *unicum*, there are no variant readings. I have standardized the spelling of *hamza* and added the corresponding *shadda* and punctuation marks. Mistakes have been corrected and the manuscript readings indicated in the critical apparatus.
The last lines of all the folios have been badly damaged by humidity. Some of them are almost completely erased, while others have been repaired with patches which hide the last two or three lines of the page completely. The guessed words appear in the edition between angle brackets. Most of the reconstructed text has been taken from similar sentences in other parts of the book; from the *Book on the Fixed Stars* by Azarquiel, on which Ibn al-Hā'im based some sections of his book; or from Ibn al-Raqqām's *al-Zīj al-Shāmil*, which reproduces verbatim some parts of our text[28]. In the last case, I have used square brackets to distinguish them. The source is indicated in the critical apparatus. Three dots between angle brackets indicate a missing or unreadable passage and between square brackets a passage dealing with unrelated subjects.

3.2. Commentary.

The commentary of the text is organized under headings dealing with the different subjects. Each comment is numbered as follows: Roman numeral of the *maqāla* (the introduction is numbered 0); number of the *bāb*; and a square bracketed number of the paragraph corresponding to the annexed edition.

[28] Ibn al-Raqqām and Ibn al-Hā'im's text are the only sources in Arabic.

The figures of the manuscript have been reproduced and extra figures have been added. Below the reproduced figures, the number identifying the folio is specified inside parentheses. As the lettering in text and figures does not always coincide, I have chosen the first ones for the reworked figures and the commentary; the figures of the text are incomplete and the lettering is often misleading.

The mathematical symbols used throughout this commentary are the following:

$\Delta\lambda$ Increase of longitude due to the trepidation motion
P_{max} Maximum accession or recession value
i Angle of rotation around the equatorial epicycle
δ Declination of a point of the equatorial epicycle
R Radius of the great circle
r Radius of the epicycles
ϵ Obliquity of the ecliptic
ϵ_{min} Minimum obliquity of the ecliptic
ϵ_{mean} Mean obiquity of the ecliptic
ϵ_{max} Maximum obliquity of the ecliptic
j Angle of rotation around the polar epicycle

3.3. Sections of the book edited and commented.

Muqaddima (fols. 3v-5v and 8v-9v), mainly on the criticisms of two of Ibn al-Kammād's books: *al-Kawr ʿalā al-dawr* and *al-Muqtabas*.

Maqāla II

Bāb 1 (fols. 23v-25v), on the accession and recession model.
Bāb 2 (fols. 26r-27r), on the parameters of the different motions.
Bāb 3 (fols. 27r-28r), on the impossibility of constructing an everlasting table for both motions.
Bāb 4 (fols. 28r-29v), on the errors found in Ibn al-Kammād's trepidation model.

Maqāla III

Bāb 1 (fol. 35v), on how to determine ϵ with the use of the tables.

Bāb 2 (fols. 35v-36r), on how to determine $\Delta\lambda$ with the use of the tables.

Maqala VII

Bāb 1 (fols. 80r-81r), on how to determine the minimum obliquity of the ecliptic from an observed obliquity.

Bāb 2 (fols. 81r-81v), on how to determine the obliquity of the ecliptic from the already known maximum and minimum obliquities.

Bāb 3 (fols. 82r-82v), on how to determine the distance between the Head of Aries and the spring equinoctial point, that is the first accession (*al-maḥsūs*).

Bāb 4 (fols. 82v-83v), on how to determine the right ascension of the degrees of the equatorial epicycle with respect to the meridian, that is the second accession (*al-ma*ᶜ*qūl*).

II Commentary

1. Ibn al-Hā'im's sources on trepidation and obliquity of the ecliptic: the "Liber de Motu" and Azarquiel's "Book on the Fixed Stars"

To understand Ibn al-Hā'im's criticisms and models, it seemed to me worthwhile to start with a brief description of the most important Andalusian models previous to Ibn al-Hā'im.

The tables found in the *Toledan Tables*[29], as well as in some of the manuscripts of the *Liber de motu octave spere*[30], are the first documentation we have of the Toledan astronomers' work on this subject.

[29] In this regard, see G.J. Toomer [1968:118-122].

[30] On the *Liber de Motu,* see the Latin text edited by J.M. Millás [1943-1950:496-509]; an English translation and commentary by O. Neugebauer [1962-290-299]; as well as the following studies: B.R. Goldstein [1964:232-247]; J. Dobrzycki [1965:3-47]; J.D. North [1967:73-83] and [1976:155-158]; and R. Mercier [1976:197-200] and [1977:33-71].

The model of the *Liber*, whose authorship is unknown although, as we have seen, it seems to have an Andalusian origin, is well known and has been studied in depth by R. Mercier [1976] and [1997].

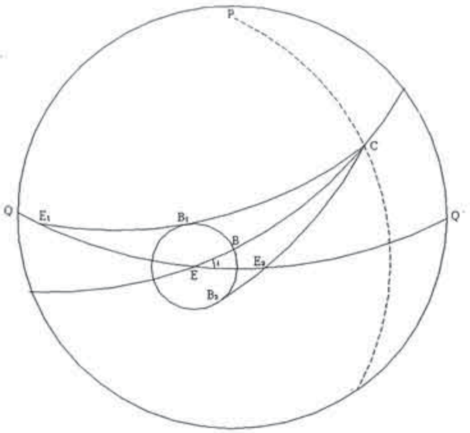


Fig. 1

In this model (Fig. 1):

QQ'	Equator.
P	Pole of the equator.
E	Intersection between the Equator and the mean Ecliptic (mean Equinox).
B, B_1, B_2, etc	Moving Head of Aries.
PC	Section of a meridian circle, the pole of which is E (EC = 90°), so that the distance between C and the equator will correspond to the mean obliquity of the ecliptic (ϵ_{mean}).
CE	Section of the Mean Ecliptic.
CB_1E_1, etc	Ecliptic, which is the great circle determined by point C and points B, B_1, B_2, etc. reaching the Equator at points E, E_1, E_2, etc.
E_1, E_2, etc	Equinoctial points, where the Ecliptic cuts the Equator.

E_1B_1, E_2B_2, etc	Arcs of the Ecliptic, which determine the increase or decrease in longitude due to the accession and recession motion ($\Delta\lambda$).

The model works as follows: on the small equatorial epicycle BB_1B_2, whose centre is E, the Head of Aries (B, B_1, B_2, etc.) moves with a uniform motion producing an angle with the equator called *i*. When point B rotates, it carries with it a moving ecliptic which always passes by point C. The intersections of this moving ecliptic with the equator are the equinoctial points (E, E_1, E_2, etc.). The increase or decrease in longitude produced by the accession and recession movement are the corresponding arcs (BE) between the different positions of the Head of Aries (B, B_1, B_2, etc.) and the corresponding equinoxes (E, E_1, E_2, etc.). The objective is to compute arc BE ($\Delta\lambda$) for any value of angle *i*. Obviously, there is another equatorial epicycle at 180° around which the moving Head of Libra rotates.

The *Toledan Tables* and the *Liber de Motu* present three tables to account for the different values of precession: a mean motion table allowing calculation of the value of angle *i* for a specific moment; and two tables which allow calculation of the increase of longitude ($\Delta\lambda$) corresponding to a particular *i* by two different procedures. The first gives directly the increase of longitude as a function of *i* while the second calculates the declination of point B, as a function of *i* and of the radius of the small epicycle, and should be used in combination with a declination table. Goldstein [1964] explains both procedures.

Azarquiel, for his part, devised three models, but concluded that the correct one was the third. This third model (Fig. 2) is basically similar to that of the *Liber* although, from a practical point of view, it introduces some important changes.

First of all, the parameters are different. Second, in the model of the *Liber* there is a fixed point, placed in a meridian at 90° from the centre of the small epicycle carrying the moving Head of Aries, through which all the different ecliptics will pass.

In contrast, in Azarquiel's third model the 90° are to be found between the moving Heads of Aries and Libra and the also moving pole of the ecliptic. And finally, the most important difference is that in his model

Azarquiel considers the movements of accession and recession and obliquity of the ecliptic to be independent, though related. This involves two different kinematic models, one for obliquity and another for accession and recession (Figs. 3 and 9).

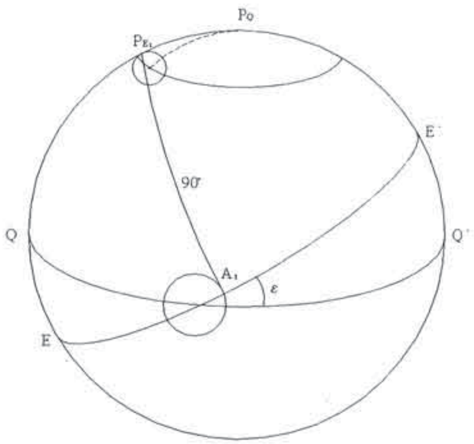


Fig. 2

Azarquiel gives three tables: the first for the mean motion of the Head of Aries (i); the second for the second accession (ZG in Fig. 9)[31]; and the third for the declination of the Head of Aries (δ).

In order to calculate the increase or decrease in longitude due to the accession and recession motion ($\Delta\lambda$), Azarquiel does not use a table like the *Liber*, but an indirect method described in his *Book on the Fixed Stars* and established by B. Goldstein[32].

[31] Called by Azarquiel the "equation of the diameter" and by Ibn al-Bannā' the "accession of the perpendicular of the diameter". On the second accession see B. Goldstein [1964:242-244] as well as chapters 12, 13 and 14 of this commentary.

[32] Goldstein [1964].

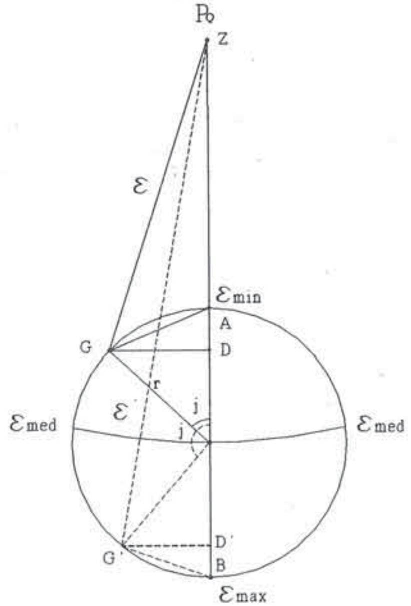


Fig. 3

Once the value of angle *i* has been determined, using the first table, similar to that in the *Toledan Tables*, a second table allows calculation of the declination of point (B, Fig. 9); then, the following formula must be used:

$$\sin \Delta\lambda = \sin \delta / \sin \epsilon$$

This means that in order to calculate the precession, the value of the obliquity must be determined beforehand.

Azarquiel's model for the obliquity of the ecliptic (fig. 3) seems to have been influenced by a model proposed by Ibrāhīm b. Sinān, involving an ecliptic, controled by an epicycle. The aim of Ibn Sinān's model was to account for the decrease in the value of the obliquity and the increase in the longitude of the apogee of the Sun[33].

Azarquiel's model (Fig. 2) consists of a deferent circle, the centre of which is the pole of the equator (P_q) with radius 23;43°, corresponding to the mean obliquity (ϵ_{mean}); and a small epicycle with radius 0;10°, the centre of which is placed on the deferent circle, around which the pole of the ecliptic (P_E) moves.

According to Goldstein, the centre of the epicycle moves along a limited arc of its deferent. Neither Azarquiel nor Ibn al-Hā'im say a single word about this specific motion, unless we consider that they are talking about it when saying that "The orb advances 8° **and then retreats the same amount and the poles of** the ecliptic orb elevate and depress 8° alternatively"[34].

At the same time (Fig. 3), the pole of the ecliptic (G), rotates about its centre with a uniform motion (*j*).

In addition, in the model of Azarquiel and Ibn al-Hā'im, the pole of the ecliptic and the Head of Aries keep a constant distance of 90°.

Since the radii are 23;43° and 0;10°, the obliquity will vary between 23;33° for Azarquiel's time and 23;53° for a moment before Ptolemy's time.

So in this model (Fig. 3) the obliquity is easily determined because it is equal to the distance between the pole of the equator and the pole of the ecliptic: that is to say GZ. So, in Azarquiel's book, the obliquity of the ecliptic is simply determined from a table rather than by the calculations that the use of the *Liber* entails[35]. This table gives the values of the obliquity as a function of angle *j* reckoned on the polar epicycle.

The revolutions in both epicycles, equatorial and polar, are in the opposite direction. Furthermore, while the Head of Aries needs 3874 Julian years to complete a revolution, the pole of the ecliptic needs only 1850 Julian years.

[33] R. Mercier [1987:106].

[34] See II.1.[1].

[35] On the procedure for calculating the obliquity according to the parameters of the *Liber*, see R. Mercier [1976:212 & 1977:39]; See also J. Samsó [1987b:367-377 & 1992:224-225] and K.P. Moesgaard [1975:97].

Later astronomers in the Iberian peninsula and the Maghrib, such as Ibn al-Kammād, Ibn Isḥāq al-Tūnisī, Ibn al-Bannā', Abū 'l-Ḥasan al-Marrākushī, Ibn al-Raqqām, Abū 'l-Ḥasan al-Qusanṭīnī, Ibn ᶜAzzūz al-Qusanṭīnī, or the astronomers of King Peter the Ceremonious, followed Azarquiel's third model. However, all of them introduced a table, similar to that of the *Toledan Tables* and the *Liber de Motu*, giving the accession or recession value (Δλ) corresponding to any value of angle *i*. The use of this kind of table is criticized by Ibn al-Hā'im.

The model presented by Ibn al-Hā'im when dealing with Ibn al-Kammād's errors is described in *Bāb* 4 of *Maqāla* II and analysed in chapter 2 of this commentary. Although the lines immediately before the description correspond to a damaged end of page and are illegible, there is no doubt that the model depicted by Ibn al-Hā'im is Ibn al-Kammād's model.

The model followed by Ibn al-Ha'im is depicted in *Bāb* 1 of *Maqāla* II and analysed in chapter 5 of this commentary.

2. On the errors detected by Ibn al-Hā'im in Ibn al-Kammād's "al-Kawr ᶜalā al-dawr" and "al-Muqtabas"

Within the rhetorical introduction, which goes from fol. 2r to fol. 10r, the author makes his first criticisms of Ibn al-Kammād. In fols. 3v to 4r the errors found in Ibn al-Kammād's trepidation model are expounded. From fol 5r to 8r, Ibn al-Hā'im describes up to 25 errors he detected in two books by Ibn al-Kammād, *al-Kawr ᶜalā al-dawr* and *al-Muqtabas*, introduced by the formula *wa-min dhālika*.

The errors are ordered and numbered at the edge of the page. Amongst them, there is the error referring to Ibn al-Kammād's model for the trepidation motion[36]. The model is compared by the author with what he calls the "Toledan observations" *(al-arṣād al-ṭulayṭuliyya)* and the works

[36] See in this respect E. Calvo [1997]; J. Chabás & B.R. Goldstein [1994] and [1996]; and J.L. Mancha [1998:1-11].

on the subject by Abū Isḥāq al-Zarqālla[37], Abū Marwān al-Istijjī[38] and Abū ᶜAbd Allah b. Barghūth[39].

As the subject is also dealt with in *Bāb* 4 of *Maqāla* II, here I will comment on the two chapters together.

0.[1] As far as the introduction is concerned, after a short foreword on the differences found between observations and calculations of the various

[37] The name of this author is spelt al-Zarqāla in the manuscript, written in Maghribī script. According to J. Samsó, this seems to be a classicised Eastern form, not documented in Andalusian sources. This information is to appear, under the entry al-Zarḳālī by J. Samsó, in a forthcoming volume of the *Encyclopédie de l'Islam*. However, al-Zarqāla is also the spelling in Ibn al-Baqqār's *Kitāb al-Adwār fī Tasyīr al-Anwār* (Escorial, ms. 418, fol 238r).

[38] Abū Marwān ᶜAbd Allah b. Khalaf al-Istijjī, according to Ibn al-Baqqār's *Kitāb al-Adwār fī Tasyīr al-Anwār* (Escorial, ms. 418, f. 242r. See also Samsó, Berrani [1999]). Abū Marwān ᶜAbd Allāh (or ᶜUbayd Allāh) b. Jalaf al-Istijī, according to Ṣāᶜid's *Ṭabaqāt*, although L. Cheikho read "al-Astuhī". Consequently, I have adopted the spelling *al-Istijjī* in spite of the fact that in the manuscript this name appears as *al-Istijibī*.

[39] This name is found as al-Faqīh al-Qāḍī Abū ᶜAbd Allāh b. Barghūt in fol. 4r and al-Faqīh Abū ᶜAbd Allāh b. Barghūth in 9v. The spelling also varies in the different editions or translations of Ṣāᶜid's *Ṭabaqāt*. Ḥayāt Bū ᶜAlwān in Ṣāᶜid al-Andalusī [3:173-6) calls him Muḥammad b. ᶜUmar b. Muhammad (b. ᶜUmar in some mss.) known as Ibn Barghūt, reading accepted by Maíllo without any explanation (Ṣāᶜid al-Andalusī [5:130], while according to R. Blachère's translation (Ṣāᶜid al-Andalusī [2:135-5]), based on Cheikho's edition (Ṣāᶜid al-Andalusī [1]), his name is Ibn Barghuth. The most recent edition of Tehran (Ṣaᶜid al-Andalusī [4]) points again to Ibn Barghuth, placing Bū ᶜAlwān's reading amongst the various errors that the author of Tehran's edition found in Bū ᶜAlwān's edition. According to Suter [1986:n. 221] the name is Muḥammad b. ᶜUmar b. Muḥammad, Abū ᶜAbd Allāh, known as Ibn Burghūth. Although Ibn Barghūth seems to be the most common spelling, in the Hebrew ms. of the *Book on the Fixed Stars* the name appears once again as Ibn Barghut. In fact, I have my suspicions about the spelling Ibn Barghūth (son of a flea). I wonder if this name, spelt as Ibn Barghūt, could be derived from the name of a well known tribe in al-Maghrib, the Barghawāṭa. We know that there was a certain relationship between this tribe and the Andalusian caliph al-Ḥakam II. Some time before the flourishing of our Ibn Barghūth (c.1050), the Barghawāṭa had sent one of its members to Cordova on a political mission (352H/963AD). Cf. R. le Tourneau [1960]. On the possible develarization of /ṭ/ in the Spanish Arabic dialect see F. Corriente [1977:40] and [1992:45-47].

motions, Ibn al-Hā'im starts his criticisms of Ibn al-Kammād's theory of trepidation found in his two books, *al-Kawr ᶜalā al-dawr* and *al-Muqtabas*, particularly the first. Ibn al-Kammād's errors led people to criticise and reject the Toledan observations after having accepted them. A group of astronomers, however, were interested in this book (*al-Kawr*) and they paid no attention to its mistakes.

Several coincidences suggest that the model attributed by Ibn al-Hā'im to Ibn al-Kammād was either Azarquiel's first model or Azarquiel's second model. First of all, according to Chabás and Goldstein[40], Ibn al-Kammād's table for the motion of the first point of Aries in *al-Muqtabas* fits fairly well with the formula underlying Azarquiel's second model. Furthermore, in the star table of the same *zīj* a precessional value for Ptolemy's epoch, found in Azarquiel's description of his first model, is used[41]. In fact, we know that these previous models were indeed used. Azarquiel himself in his two treatises on the construction and use of the equatorium, used his first model to adapt al-Battānī's planetary apogees to his epoch[42].

However, the fact that Ibn al-Kammād's model, as described by Ibn al-Hā'im, presents the polar and equatorial epicycles, like Azarquiel's third model, rules out the abovementioned possibility. In my opinion, Ibn al-Kammad's model is just a misinterpretation of Azarquiel's third model.

0.[2] Ibn al-Hā'im goes on to say that he has seen a copy of the book (*al/Kawr*) with a note written in one of the aforementioned astronomers own hand, in which he proves his ignorance by praising Ibn al-Kammād's trepidation table with the argument that it is Azarquiel's table, although in fact it is not. Ibn al-Hā'im marveles at the ignorance showed by this man.

0.[3] In fact, we find here a quotation from Azarquiel's *Book on the Fixed Stars* about the impossibility of constructing a table that gives

[40] J. Chabás & B. Goldstein [1994:24].

[41] See M. Comes [1991].

[42] M. Comes [1990:88-92].

directly the value for $\Delta\lambda$ because of the changing obliquity. The quotation corresponds word for word with the last paragraph of Azarquiel's 8th chapter[43]. Ibn al-Hā'im expands his criticisms on the use of such a table in 0.[13] and II.3.[4].

0.[4] According to Ibn al-Hā'im, an otherwise unknown astronomer, a contemporary of his, prepared a *zīj* named *al-Zīj al-Muntakhab*, which reproduced Abū Marwān al-Istijjī's mean motion tables and al-Battānī's equation tables. In this *zīj*, Ibn al-Hā'im also finds the bulk of errors he criticized in Ibn al-Kammād's *al-Kawr ᶜalā al-dawr*. However, as we will see[44], the parameters implied in *al-Kawr ᶜalā āl-dawr* and in the *Muntakhab zīj*, at least as far as the trepidation tables are concerned, seem to be unrelated. This *zīj* appears also mentioned in 0.[11]. Unfortunately, we have no references to the author of this *zīj* nor to the *zīj* itself.

Following this, there is an excursus, not edited here, on the errors found in the *Muntakhab zīj* related to the motion of the sun. In it we find another quotation on the solar motion from Abū Marwān's *Risālat al-Iqbāl wa-l-idbār*, which has not survived. Since this book is lost, and given the exactitude of the previous quotation of Azarquiel, I think that this quotation deserves to be studied within the framework of the Andalusian models for the motion of the sun[45].

0.[5] The next step will be to discuss the errors in the trepidation motion: firstly, Ibn al-Kammad's errors in *al-Kawr ᶜalā al-dawr*; and secondly, the mistakes found in the *Muntakhab zīj*[46].

0.[6] Ibn al-Kammād's errors on this subject, appearing in fols. 5r to 8r, are divided into four aspects (*jihāt*).

[43] J.M. Millás [1943-1950:335].

[44] See the commentary to 0.[10].

[45] On this see E. Calvo [1998].

[46] The errors in *al-Zīj al-Muntakhab* will be discussed in chapter 3 of this commentary.

First: Ibn al-Kammād's premises are two: that the motion of the pole of the ecliptic starts from one of its mean values, while the motion of the Head of Aries starts from the equator, and that the two motions are equal. See Fig. 4 described in II.4.[2].

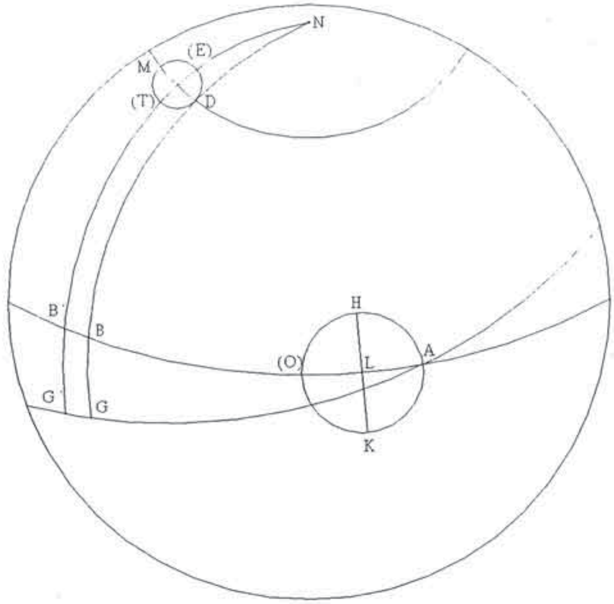


Fig. 4

0.[7] Second: Ibn al-Kammād devised a table for the two motions, which he considered valid for any time, and structured as an equation table for arguments between 1° and 90°. According to Ibn al-Hā'im this was not correct, because for this range of arguments of the motion of the pole of the ecliptic around the polar epicycle the pole does not achieve all of its obliquities. To go from the maximum to the minimum obliquity, the pole needs not only to cover 180° but also to meet the conditions explained in 0.[8].

0.[8] Third: The problem posed by Ibn al-Hā'im in this rather obscure paragraph is that even if the table was symmetrical for 180°, the problem remains. The pole of the ecliptic should not be placed at ϵ_{mean}, because in this case, from one to the other ϵ_{mean}, the pole would not complete all its possible obliquities, from maximum to minimum. To achieve this, the pole should be placed at its minimum or maximum distance from the pole of the equator (ϵ_{min} or ϵ_{max}), but, in this case, the Head of Aries would be at the Northern or Southern limits of the equatorial epicycle and not at the intersection with the equator, where $\Delta\lambda$ is 0° and the Head of Aries coincides with the vernal equinox, as proposed by Ibn al-Kammād.

0.[9] Fourth: Finally, Ibn al-Hā'im's last criticism is that the values attributed to trepidation by Ibn al-Kammād do not coincide with the values observed, which are the starting point (*aṣl*) used by the Toledan school to determine the parameters of the trepidation model.

Table 1

		Azarquiel	Ibn al-Hā'im's *uṣūl*
Hipparchus	$\Delta\lambda$	9;28,30°	9;29°
	i	9^s 22;32,12°	9^s 22;32,40°
Ptolemy	$\Delta\lambda$	6;42,45°	6;42°
	i	10^s 19;1,30°	10^s 19;2°

Table 1 summarizes the values of the $\Delta\lambda$ and the argument for the Head of Aries (*i*) for the times of Hipparchus and Ptolemy, as reported by Azarquiel in his *Book on the Fixed Stars*, and the values mentioned by Ibn al-Hā'im.

As we can see in this table, Ibn al-Hā'im's values are very close to those of Azarquiel's third model.

Entering with these values of *i* Ibn al-Kammād's equation table, Ibn al-Hā'im obtains the following results:

	$\Delta\lambda$	dif. with the *uṣūl*
Hipparchus	9;19°	- 0;10°
Ptolemy	6;57°	0;15°

That is to say, a difference of up to fifteen minutes between the stated[47] values (*aṣl*) and the ones obtained using Ibn al-Kammād's tables in his *zīj al-Kawr ʿalā al-dawr*. Using the tables in the *Muqtabas zīj*[48], I obtained the same values, that is 9;19° for Hipparchus and 6;57°[49] for Ptolemy.

II.4.[1] *Bāb* 4 of *maqala* II is also devoted to Ibn al-Kammād's errors on this subject. In the first paragraph -hard to read because it coincides with the damaged end of the page- Ibn al-Hā'im just states his two main criticisms of Ibn al-Kammād's trepidation model: the similarity of the two motions, and the errors in the construction of the table.

II.4.[2] The model is depicted in fol. 29v and reproduced in Fig. 4. Ibn al-Kammād's model corresponds roughly to Azarquiel's third model and that of the *Liber de Motu*[50]. The description in the text is as follows:

AB	arc of the equator (90°)
AG	arc of the ecliptic (90°)
AHK	equatorial epicycle
D<T>M<E>[51]	polar epicycle
NDBG	solstitial colure

[47] In the text, these values are quoted as "observed".

[48] Biblioteca Nacional de Madrid, ms. n. 10023.

[49] In the manuscript the value is 7;57°. However, this value should be corrected to 6;57° because 7° has to end at 10^{s} 18° and not at 10^{s} 20° as it appears in the manuscript due to a displacement in the column of the degrees. In fact displacement errors are fairly common in this manuscript. This value is found corrected by Samsó in [1997:109], as it appears in the *Muwāfiq Zīj*, which offers the same table. In the table reproduced by Chabás & Goldstein [1994:25], the value is not corrected, although the correct value appears in the variant readings taken from the *Barcelona Tables* (Ms. Ripoll 21, fol. 137r).

[50] See chapter 1 of this commentary.

[51] Points T and E are not mentioned in the description of the model, although they are used in the rest of the text. See II.4.[4].

N	pole of the equator
D	pole of the ecliptic

In this model, the figure of 90°, which has created many problems of interpretation from the very beginning, is to be taken on the equator and the ecliptic, between the Head of Aries and Libra at the beginning of their motions and the colure. In both Azarquiel's third model and Ibn al-Hā'im's, the 90° are taken between the moving Head of Aries and the moving Pole of the ecliptic. In the *Liber de Motu* they are considered to be between the centre of the equatorial epicycle (mean first point of Aries) and a fixed point in the colure, which corresponds with the intersection between the moving and fixed ecliptics. This was the assumption made by North[52], followed by Samsó[53], and afterwards accepted by Mercier as the best one. Regiomontanus' model, however, coincides with Ibn al-Hā'im's description of Ibn al-Kammād's model[54].

II.4.[3] We find here the repetition of Ibn al-Kammād's opinion. The Head of Aries and the pole of the ecliptic would cover equal arcs in equal times. The Head of Aries starts from the point corresponding to the intersection between the ecliptic and the equator and the pole of the ecliptic starts from its mean distance from the pole of the equator (ϵ_{mean}).

II.4.[4] This paragraph aims to show once again that a table for 90° would be enough for the motion of the Head of Aries, because the variable used to compute the table is the declination of the Head of Aries (δ), which

[52] See J. North [1967:71-83] and [1976:155-158]. In the first paper, North presents the constraint, afterwards accepted by Mercier [1996:305], according to which the 90° should be taken between the centre of the equatorial epicycle and the solsticial colure. In the second, he seems to accept Mercier's previous and subsequently abandoned opinion, according to which the Head of Aries should be considered instead of mean Aries. In this case, the model would have been similar to that used by Ibn al-Kammād.

[53] See J. Samsó [1992:222-224].

[54] See R. Mercier's account [1996;305-6] and J. Samsó [1992:222].

goes from a minimum value for $i=0°$ to a maximum value for $i=90°$[55].

However, to compute a table for obliquities, we need 180° because the minimum and maximum values are for $j=0°$ and $j=180°$. But, even if the table was designed for 180°, the error of starting the motion of the pole from its ϵ_{mean} remains, because from first to second ϵ_{mean}, the pole does not pass through both ϵ_{max} and ϵ_{min}, but only through one of them. The text considers that computing a table using the arc TM to complete the arc DT has no utility (*fā'ida*) because TM is not an adequate couple (*zawj*) for DT due to the fact that the values for the obliquity in both arcs are symmetrical. To calculate a table for the obliquity of the ecliptic for a range of arguments of 180°, a second condition is necessary: to add arc ED (instead of TM) to arc DT.

II.4.[5] Then, the Head of Aries will go back to the place from which the motion was initiated and reach point K, while the pole of the ecliptic will have moved to point E, which corresponds to the minimum obliquity of the ecliptic (ϵ_{min}).

II.4.[6-7] To deal with this problem Ibn al-Hā'im suggests a solution that he had already proposed in the introduction: to place the beginning of the motion of the Head of Aries at its maximum southern declination and, therefore, the beginning of the motion of the pole of the ecliptic at its ϵ_{min}, i.e. to compute a table either from E to T and from K to H or from T to E and H to K (Fig. 4). However, this would be correct only if the two motions were the same, which, according to Ibn al-Hā'im is not the case, as he has already shown in II.1 to 3.

[55] The terminology is, apparently, confusing. The text says that after having moved through an arc of 90°, the Head of Aries will have attained "all its maximum declinations from the equator" (*jamīᶜ muyūli-hi al-kulliyya ᶜan muᶜaddil al-nahār*). *Al-mayl al-kullī* is the obliquity of the ecliptic but in this case I assume it to mean the declination of the Head of Aries. There is also the possibility that Ibn al-Ha'im was trying to say that in fact, according to the relationship he will show in II.3, during these 90° the ecliptic would have attained all its values for the obliquity. Certainly the ratio between the revolution period of the pole of the ecliptic and that of the Head of Aries is not 2/1 (1;54,40^p/1^p) but the difference as far as the table is concerned will be negligible.

2.1. A further commentary on the errors that Ibn al-Hā'im attributes to Ibn al-Kammad

The criticisms we have just seen need a more extensive commentary. We know of a reference to the fact that Ibn al-Hā'im had detected various errors in Ibn al-Kammād's trepidation models. This reference is found in the *Natā'ij al-afkār fī sharḥ Rawḍat al-azhār*, one of the commentaries on the *Rawḍat al-azhār fī ʿilm waqt al-layl wa-l-nahār* (794/1391-92), the *urjūza* on timekeeping written in Fes by Abū Zayd ʿAbd al-Raḥmān al-Lakhmī al-Jādirī (777-818/1375-1416)[56]. There are two anonymous copies dated 1183/1770, although the *annus praesens* is 920/1515[57], and another copy, undated but probably written around the end of the 16th century, in which the name of the author appears as Abū Zayd ʿAbd al-Raḥmān al-Jānātī al-Nafāwī[58]. Most of the data in it are expanded on in the *Kanz al-asrār wa-Natā'ij al-afkār fī sharḥ Rawḍat al-azhār* by Abū'l-ʿAbbās al-Māwāsī al-Fāsī (d. 911H /1505). However, the reference to Ibn al-Hā'im and Ibn al-Kammād is not found in the latter work. These commentaries contain some interesting remarks which indicate that the trepidation theory was no longer applicable[59].

Until recently, we had no documentation that confirmed the error attributed to Ibn al-Kammād by Ibn al-Hā'im. However, in a manuscript in the Cathedral of Segovia[60] there is a medieval Castilian translation of a chapter by Ibn al-Kammād dealing with trepidation, which gives a full

[56] Ms. 80 in the Maktabat al-Zāwiyya al-Ḥamzawiyya (Ayt Ayache), fols. 203-220. This manuscript is partially described in M. Mannūnī [1963], n. 157. A full description of it is to be found in A. Alkuwaifi & M. Rius [1998].

[57] On ms. Cairo K 4311, see D. King [1981];[1986]. See also ms. London British Library Or 411.

[58] Maktaba Ḥamzawiyya 80, 228-334.

[59] See J. Samsó [1998] and M. Comes [1997].

[60] Segovia, Biblioteca de la Catedral ms 115 fols. 218vb-220vb. "Yuçaf Benacomed Libro sobre çircunferençia de moto". See J.L. Mancha [1998], where the edited text appears in pp. 8-10.

explanation of the identity of the two motions. Its title, *Libro sobre çircunferencia de moto sacado por tiempo seculo*, seems to be a translation of *al-Kawr ʿalā al-dawr* and/or *al-Amād ʿalā al-abad*.

The idea, stated in the Cathedral of Segovia manuscript, is the one we have seen criticized by Ibn al-Hā'im. The motion of the pole of the ecliptic around its polar epicycle is equal to the motion of the Head of Aries around its equatorial epicycle. Furthermore, the former will start at its mean value (ϵ_{mean} = 23;43°), that is when the pole of the ecliptic is either on M or D, between its nearest distance (T) and farthest distance (E) to the pole of the equator, and the latter will start when the Head of Aries is at the equator (A) and the declination is 0°.

Hence, accession will result from the motion of the Head of Aries in one half of the epicycle, while recession would occur in the other half and, at the same time, the obliquity of the ecliptic will increase as its pole rotates from the midpoint of one half of its epicycle to the other midpoint, and will decrease in the other half.

When talking about the changing obliquity of the ecliptic and the motion of the equinoxes, in his *Tadhkira*, Al-Ṭūsī introduces a difficult to understand paragraph, that and has been edited and translated by J. Ragep[61] as follows:

وذهب بعضهم إلى الاكتفاء بمحرك واحد للاختلافين يحرك فلك البروج فتتحرك كل نقطة منه حول دائرة صغيرة فيكون من الحركة في أحد نصفيه الإقبال ومن الحركة في النصف الآخر الإدبار ومن الحركة من منتصف أحد النصفين إلى منتصف النصف الآخر انتقاص الميل ومن الحركة في النصف الآخر ازدياده.

"One [some?] of them came to be satisfied with one mover for both divergencies. This mover would cause the ecliptic orb to move in such a way that every **point** on it moves about a small circle. Accession would then result from the motion in one of its halves, while recession would occur in the other half. [In

61 J. Ragep [1993:II.4[5]].

addition], there would occur a decrease in the obliquity during the motion from the midpoint of one of these halves to the midpoint of the other half, while there would be an increase during the motion in the other half".

However if we replace a single word, ***nuqṭa*** (**نقطة**) **"point"** with ***quṭb*** (**قطب**) **"pole"**, two words that are easily confused in Arabic, and of course reading the verb as masculine instead of femenine, we have exactly the same idea found in Ibn al-Kammād and criticized by Ibn al-Hā'im. It is then clear that al-Ṭūsī is describing Azarquiel's model, but in Ibn al-Kammād's version. Al-Ṭūsī also specifies that this is not his own opinion, but the opinion of other people. This is usual in the *Tadhkira*, where al-Ṭūsī often introduces ideas and opinions that he does not share in an attempt to present everything he knows on a particular subject.

There are some other indications that al-Ṭūsī knew about Azarquiel's trepidation and obliquity of the ecliptic models, and that he had access to materials coming from Ibn al-Kammād. In a recent communication in Tehran[62] I showed the existence of many points of contact between the Marāgha astronomers and those of the court of Alfonso X, amongst them those related to Azarquiel. I will summarize them here:

1) J. Ragep[63] notes that in Theon's theory of trepidation the solstitial points are claimed to move, while al-Ṭūsī modifies the theory as reported by Theon in order to make it compatible with his general cosmological perspective, in which it is the vernal equinox that defines the reference system. To this we should add that, once again, al-Ṭūsī attributes this modification to "some of the practitioners of this discipline". Unlike Eastern trepidation models, such as the models of Ibn Sinān or al-Khāzin[64], in the Andalusī trepidation models derived from Azarquiel's models, the vernal equinox is precisely the reference point.

[62] See M. Comes [1998].

[63] J. Ragep [1993:Commentary II.4 [1]].

[64] See Introduction 1.

2) In his *Tadhkira*[65], al-Ṭūsī refers to a "maximum and minimum" value for the obliquity that immediately recalls Azarquiel's model, especially if we take into account that according to al-Ṭūsī, the maximum should be less than 24° and the minimum not less than 23;33° and Azarquiel's model gives a maximum of 23;53° and a minimum of 23;33° precisely. Other authors defend a decreasing value, but Azarquiel is the first one to give a model with a maximum and minimum parameters.

3) J. Ragep also suggests the possibility that al-Ṭūsī was aware of the work of Azarquiel when he mentions an apparent recognition in the *Tadhkira* that the solar apogee may have its own motion[66]. In fact Ibn al-Shāṭir (Damascus, d. 1375) maintained that according to his observations the solar apogee moved at a different rate from precession, an opinion that coincides with Azarquiel, and is at variance with his own contemporaries who thought that the solar apogee moved at the same speed as precession. He also demonstrates his knowledge of the trepidation theory and the corresponding models although he dismisses them because they do not accord with the observations[67].

3. On the errors detected by Ibn al-Hā'im in the "Muntakhab zīj"

0.[10] Ibn al-Hā'im will show now how to test the errors found in the *Muntakhab zīj*, in which, according to him, Abū Marwān al-Istijjī's mean motions were used some 150 years later by the aforementioned contemporary of Ibn al-Hā'im.

Neither the *Muntakhab zīj* nor Abū Marwān's *zīj* are extant. However, in his *Risālat fī al-Tasyīrāt wa-Maṭāriḥ al-Shuᶜāᶜāt*[68] Abū Marwān suggests the use of a certain *zīju-nā* ("our *zīj*"), which could be either a *zīj* of his own, and the source for the abovementioned *Muntakhab zīj*, or the

[65] J. Ragep [1993:II.4 [1]].

[66] J. Ragep [1993:Commentary II.6 [1]].

[67] See G. Saliba [1994:235] and M. Comes [1997].

[68] Ms. Escorial n. 939, fol. 12r. See J. Samsó & H. Berrani [1999:296-298].

Toledan Tables. However, as we will see in this chapter, Abū Marwān's data, as quoted by Ibn al-Hā'im, do not agree with the trepidation parameters in the *Toledan Tables*.

Ibn al-Hā'im uses the following procedure to determine that the trepidation tables in the *Muntakhab zīj* are not correct.

First, he gives the data on which Abū Marwān based his trepidation model. Although he does not say so explicitly, I assume that Ibn al-Hā'im is referring here to the model described by Abū Marwān in his *Risālat al-Iqbāl wa-l-Idbār*, previously mentioned by Ibn al-Hā'im[69], and which is not extant either.

Abū Marwān's values are different from Azarquiel's and Ibn al-Kammad's and it seems that he probably used his own model, which differs from all of Azarquiel's three models: neither Δλ nor *i* coincide with the data given by Azarquiel for his first and second models[70]. The data are reproduced in Table 2.

Table 2

		Azarquiel[71]	Abū Marwān
Hipparchus	Δλ	9;28,30°	9;38,40°
	i	9^s 22;32,12°	9^s 23;42°
Ptolemy	Δλ	6;42,45°	6;50,40°
	i	10^s 19;01,30°	10^s 19;23°

Entering with i in the tables in the *Muntakhab zīj*, Ibn al-Hā'im obtains a difference of about five minutes between Abū Marwān's precession value and the ones derived from the aforementioned *zīj*. The values and differences are the following:

69 See 0.[4].

70 On these parameters see J. Samsó [1994:VIII/9-27].

71 In Azarquiel's third model.

Δλ	*Muntakhab*	Abū Marwān	dif.
Hipparchus	9;45,30°	9;38,40°	0;6,50°
Ptolemy	6;55,15°	6;50,40°	0;4,35°

This information requires a few comments:

1) The values obtained by Ibn al-Hā'im using the tables in the *Muntakhab zīj* are close to the ones of the second approach in Azarquiel's 2nd model:

Δλ	Azarquiel	*Muntakhab*	dif.
Hipparchus	9;45°	9;45,30°	0;0,30°
Ptolemy	6;57°	6;55,15°	0;1,45°

As Ibn al-Hā'im states that the *Muntakhab zīj* contained the bulk of errors found in Ibn al-Kammad's *al-Kawr ʿalā al-Dawr*[72], I have also recalculated the Δλ, using the tables in *al-Muqtabas*. The values achieved (c. 9;16,24° for Hipparchus and c. 6;53,9° for Ptolemy) show that the tables in the two *zījes* are different.

2) As regards the increase of precession, determined by the estimated difference between the two positions of *Qalb al-Asad* for Hipparchus and Ptolemy's times, we have the following differences:

	Az. s.p & 1B[73]	Az. 2B[74]	Abū Marwān	*Muntakhab*
Δλ	2;47°	2;48°	2;48°	2;50,25°

[72] See the paragraph 0.[4].

[73] The difference is based on the determination of the longitude of *Qalb al-Asad* used by Azarquiel as starting point (s.p.) for his study of precession. 1B stands for Azarquiel first model, second variant. See Samsó [1994:7, 26].

[74] 2B stands for Azarquiel second model, second variant. See Samsó [1994:26]

According to J. Samsó[75], the fact that for Abū Marwān the increase in the value of precession between Hipparchus' and Ptolemy's times was 2;48°, very close to the difference between the corresponding longitudes of the star *Qalb al-Asad* (2;47°) determined by Azarquiel, also seems to point to a relationship between the two authors. And, in fact, as we can see, the difference between Hipparchus' precession and Ptolemy's in the second approach of the second model gives us exactly 2;48°. However, the difference derived from the values obtained using the tables in the *Muntakhab zīj* is 2;50,25°.

3) As Samsó & Berrani[76] suggested that Abū Marwān could be one of the authors of the *Toledan Tables*, I have tried to enter its trepidation tables with *i* as argument. The result gives precession values which do not agree with the ones that, according to Ibn al-Ha'im, proceed from Abū Marwān.

$\Delta\lambda$	*Toledan Tables*	Abū Marwān	dif.
Hipparchus	9;49,30°	9;38,40°	0;7,52°
Ptolemy	6;58,33°	6;50,40°	0;10,50°

4) As far as P_{max} is concerned and to recompute the table which supplies the increase or decrease of precession in the *Toledan Tables*, I have used the following approximation[77]:

$$\sin P_{max} = \sin \Delta\lambda \, / \, \sin i$$

From Abū Marwān's parameters, I derived a P_{max} of 10;32,53° (from Ptolemy's values) and 10;32,32° (from Hipparchus'). These values are also different from P_{max} = 10;45°, which appears in the *Toledan Tables* and the *Liber de Motu*.

[75] See J. Samsó [1994a:28].

[76] See J. Samsó & H. Berrani [1999:296-298].

[77] See J. Samsó [1994a:4].

From the values Ibn al-Hā'im obtained using the tables in the *Muntakhab zīj*, I derived the following maximum increase of precession:

$$P_{max} = 10;40,0,37°(P) / 10;40,1,54°(H).$$

Surprisingly enough, 10;40° is the P_{max} found in Ibn al-Raqqām's *al-Zīj al-Qawīm* and in Abū 'l-Ḥasan al-Qusanṭīnī[78].

Using Ibn al-Raqqām's table, the values obtained and the difference between these values and the values obtained by Ibn al-Ha'im using the *Muntakhab zīj* are as follows:

	Δλ Ibn al-Raqqām	Δλ *Muntakhab*	dif.
Hipparchus	9;45,30°	9;45,30°	0;0,0°
Ptolemy	6;55,56°[79]	6;55,15°	0;0,41°

As a conclusion, we can say that Abū Marwān's parameters differ from the other known parameters, although his determination of the increase of precession between Ptolemy and Hipparchus coincides with Azarquiel's.

Furthermore, his parameters are not related to those in the *Toledan Tables* or the *Liber de Motu*, although this does not rule out his intervention in the preparation of the *Toledan Tables*.

The parameters for Hipparchus' and Ptolemy's precession in the *Muntakhab zīj*, however, are very similar to those of Azarquiel's second approach in the second model.

Furthermore, it seems that the table for determining the increase or decrease of precession in this *zīj* may have coincided with the table found in Ibn al-Raqqām's *al-Zīj al-Qawīm* and in Abū 'l-Ḥasan al-Qusanṭīnī, which also differ from the tables found in the rest of Azarquiel's followers.

[78] See M. Comes [1996:360].

[79] It seems that there is a misreading here, either between 56" and 15" or previously in the value of *i*.

4. On the construction of correct tables for this motion[80]

0.[11] Ibn al-Hā'im computed [planetary] positions using al-Battāni's equation tables and the value of precession calculated using the tables in the *Muntakhab zīj*. However, the observations of meridian transits (*al-majāzāt al-istiwā'iyya*) did not agree with the positions calculated. There are major differences between computation and observations in the periods of time between two meridian transits. Such differences cannot be attributed to observational errors. This is the kind of error one finds in modern books on the subject. Ibn al-Hā'im states that somebody, whom I cannot identify, but who according to him was a man who tried to reach truth through both theory and practice, warned him of this.

In fact, Ibn al-Hā'im states that he has checked this trepidation motion by using it in combination with al-Battānī's planetary equations. This appears to be a reference to the *Muntakhab zīj* which, according to 0.[4], contained al-Istijjī's mean motion tables and al-Battānī's planetary equations. It is logical to imagine that al-Istijjī's mean motions were sidereal and that the computation gave sidereal longitudes to which precession should be added, using trepidation tables of some kind. The only information we have on these trepidation tables appears in 0.[10], where, as we have seen, Ibn al-Hā'im states that the values of trepidation obtained with the tables of the *Muntakhab zīj* did not agree with the *uṣūl* used by al-Istijjī. We may wonder whether the mean motion of the Head of Aries in the *Muntakhab zīj* was computed with a mean motion table copied from al-Istijjī. On the other hand, in my commentary to 0.[10] I have also suggested that the table to calculate the equation of trepidation in the *Muntakhab zīj* might be the one we find in Ibn al-Raqqām's *Qawīm Zīj*.

0.[12] Taking all this into account, Ibn al-Hā'im decided to devise his own tables for the trepidation motion, after having studied what the followers of this motion had said before him, as well as the errors made. He creates a table called *Jadwal al-juyūb li-mayl Ra's al-Ḥamal*. The variables used are the following:

[80] On the possibility that the *Kāmil Zīj* had tables, see M. Abdulrahman [1996a].

i Angle of rotation around the epicycle.
δ Declination of a point of the epicycle from the equator.
r Radius of the epicycle.
$\Delta\lambda$ Accession or recession.
ϵ Obliquity of the ecliptic.

This table seems to be similar to the table found in Azarquiel, which gives δ. However, the word *juyūb* appears explicitly in the title and hence this table, which is not extant, would give "sin δ"[81] instead of "δ".

So, Ibn al-Hā'im would have a table giving directly the sine of the declination (sin δ). This table is also found in Ibn al-Raqqām's *al-Zīj al-Shāmil*[82], composed to provide tables for Ibn al-Hā'im's book, as stated by the author himself.

In fact, what he needs is to determine is $\Delta\lambda$ (BD in Fig. 8). Hence, it is not necessary to compute *arcsin (r sin i)* and devise a table for δ, as Azarquiel does. The reason is that to calculate the accession and recession value with the formula $\sin \Delta\lambda = \sin \delta / \sin \epsilon$, he just needs $\sin \delta$, which is calculated according to the formula $\sin \delta = r \sin i$, used by Azarquiel[83] and Ibn al-Hā'im.

0.[13] Once he has determined $\sin \delta$, he devises a table to calculate $\Delta\lambda$ directly, but only for the accession motion, which occurs in the northern side of the equator and between 0^s and 6^s. Ibn al-Hā'im explicitly states that the table can be used for the second half of the equatorial epicycle, that is, for recession, because in that case the errors would amount only to seconds of arc. However, as this table will not be useful for a second revolution, he recommends devising a new table using the sines of the new obliquities.

The explanation given by Ibn al-Hā'im, based on the relationship between the revolution periods of the two motions, the motion of the Pole

[81] This is confirmed by VII.3.[1].

[82] See E.S. Kennedy [1997:56].

[83] M. Comes [1996:349-364].

and that of the Head of Aries, is found in II.3.[1-3][84].

This passage is, then, only apparently contradictory: the motion of the Head of Aries through an arc of 180° will take a long period of time (c. 2000 years) during which there will be important changes in the obliquity of the ecliptic. The only explanation is that Ibn al-Hā'im's equation table takes into account the value of ϵ for each mean position of the Head of Aries on the equatorial epicycle and, hence, using a different ϵ for each value of i and applying an expression of the type:

$$\sin \Delta\lambda = r \sin i / \sin \epsilon$$

In fact, he will establish below (II.3.[1-3]) that while the Head of Aries moves through an arc of 180°, the pole will have moved through 180° × 1;54,40°, that is 344°. This implies that the values for arguments between 180° and 360° will not be symmetrical to those between 0° and 180°. The error will be small, but it will increase considerably when the Head of Aries has completed a revolution, while the pole of the ecliptic will have moved 688°: equivalent to two revolutions minus 32° (see below II.[3].

0.[14] Here there is a rhetorical paragraph, not edited, in which the author acknowledges the indebtedness of his work on this motion to Azarquiel's models. He outlines his exhaustive investigations and praises the skill of Azarquiel and the excellence of his models and books.

Ibn al-Hā'im talks here about *al-ḥarakāt al-mustadraka ᶜalā al-qudamā'*. The term *al-mustadraka*, which also appears when dealing with the lunar model[85], seems to have been used in al-Andalus to refer to the objections made to different assertions of the ancients, mainly Ptolemy. In fact, Saliba[86] has followed the use of this term in al-Andalus. He mentions a lost treatise entitled *Kitāb al-Istidrāk ᶜalā Baṭlamiyūs*, by an unknown contemporary of Azarquiel. This book seems to be devoted to expounding

84 See chapter 7 of this commentary.

85 *Al-ziyāda al-mustadraka ᶜalà al qudamā'*. See R. Puig [2000:76,91].

86 For the implication of this term, see G. Saliba [1996:83-86 & 1999:3-25].

the various problems that Ptolemy's astronomical theories posed.

Ibn al-Hā'im then introduces the observations of different astronomers of the position of *Qalb al-Asad*, the star most used for this purpose, due to its proximity to the ecliptic.

Table 3
Positions of *Qalb al-Asad*

	Ibn al-Hā'im	Azarquiel1	Azarquiel2
Hipparchus	3^s 29;50°	3^s 29;39°[87]	3^s 29;50°
Ptolemy	$<4^s>$2;30°	4^s 2;26°[88]	4^s 2;35°[89]
al-Battānī	4^s 14;00°	4^s 13;58°[90]	---------
Ibn Barghūth	4^s 16;20°[91]	------------	4^s 16;20°[92]

Table 3 shows the values found in this paragraph and Azarquiel's values from his *Book on the Fixed Stars*.

In this table, *Azarquiel1* corresponds to Azarquiel's calculations based, according to Azarquiel himself, on Thābit b. Qurra's "observed"[93] values,

[87] Probably this value should read 3^s 29;49°, and the error comes from taking 3;30° instead of 3;40° as the basis for calculation for the solar apogee position related to the position of *Qalb al-Asad*, between Hipparchus' and Thabit's epochs. In fact 3^s 29;50° is a well known value.

[88] The value in the *Almagest* 2;30°. See Toomer [1984:367] and Kunitzsch [1986:94-95;266-267].

[89] This position is approximate and comes from deducting fom Azarquiel's position (c. 136;35) a difference established between Azarquiel and Ptolemy of 14°.

[90] The value in al-Battānī's star table is 14°. See Nallino [1899:258].

[91] 16° in words and 16;20° in *abjad*.

[92] Ibn Yūnus (1032) determined a value of 4^s 16;19°.

[93] If Azarquiel took this value from Thābit's *Tract on the Solar Year*, we are dealing here with Ma'mūnī's observations, as stated by Morelon [1994:132-133] and Samsó [1994:VIII,8]. However the source is not explicitly stated in Azarquiel's *Book on the*

and taking into account the position of the solar apogee[94]. *Azarquiel2* corresponds to the supposed observations of the different astronomers as quoted by Azarquiel[95].

Table 4 shows the increase in time between the various attested or calculated observations.

Table 4

Δ of time between the various attested positions of *Qalb al-Asad*

	Δt acc. Ibn al-Hā'im	Δt acc. Azarquiel
Hipparchus	-----	-----
Ptolemy	287 from Hipparchus	285 id.
al-Battānī	741[96] from Ptolemy	744 id.
Ibn Barghūth	171 from al-Battānī	167 id.

In this table, the increase in time between the supposed observations quoted by Azarquiel in his *Book on the Fixed Stars* is not explicitly stated and has been deduced by researchers[97].

According to Azarquiel, the dates of the observation were the following: Hipparchus 150 BC; Ptolemy 137 AD; al-Battanī 883 AD; Ibn Barghūth 1049-1050 AD. However, Azarquiel is confusing. He gives the data in Arabic years but the increase in time between observations in Julian years,

Fixed Stars, and Azarquiel not only adds a paragraph praising Thābit's skill and method as an observer but also gives a value (4^s 13;13°) that is different from the value found in Thābit's *Tract* (4^s 13;2°) and from that of Ḥabash and al-Farghānī (4^s 13;15°) (Girke [1988]). All this suggests that Azarquiel may have been using a different source.

94 J.M. Millás [1943-1950:297-8].

95 J.M. Millás [1943-1950:305,309-314].

96 In the text the value is 941, which most probably corresponds to a confusion between 700 and 900, which is very usual in Arabic script.

97 I have used J. Samsó [1992:236]. R. Mercier [1996:328] calculates the following observation times: Hipparchus 145BC; Ptolemy 140AD and al-Battanī 884AD, which gives intervals of 285 years and 744 years between the aforementioned astronomers.

although this is not always stated in the text.

To devise the third model in the *Book on the Fixed Stars*[98], Azarquiel used the positions of *Qalb al-Asad* for different times, deduced from Thābit b. Qurra's values[99], taking into account the motion of the solar apogee, as was studied in chapter one of his lost solar treatise. According to him, these deduced values are more reliable that the observed ones, because he hold Thābit to be more reliable than anyone else.

As we can see, Ibn al-Hā'im's values are exactly the values attributed by Azarquiel to the different astronomers, except for al-Battanī's, which is not in *Azarquiel2* but is a rounding off of *Azarquiel1*. The small differences in the intervals of time probably correspond to the data from which the calculations had been deduced.

At the end of chapter 5, Azarquiel determines that the star *Qalb al-Asad* was at Leo 9;8° at the moment when $\Delta\lambda = 0°$. This is the value found in all the following Andalusī and Maghribī star tables entitled *jadwāl al-kawākib min al-mabdā' al-dhātī* and calculated for that moment. The difference between this position (Leo 9;8°) and the position of *Qalb al-Asad* determined by Ptolemy (Leo 2;30°) will give us the increase of 6;38° on Ptolemy's star positions found in all these tables[100].

0.[15] The motion affecting this star is what is called motion of accession. The positions of the star indicate that the orb was in accession (*muqbila*) at these times. Finally, there is a long excursus, not edited in the appendix, on the difficulties the other astronomers had found in understanding this motion correctly, which ends with the explanations of the different reasons why the author entitled his book *al-Zīj al-Kāmil*.

98 J.M. Millás [1943-1950:295-300].

99 On this see J. Samsó [1994:7-8].

100 Goldstein & Chabás disagree with this; they maintain that these tables are calculated for the beginning of Hijra and are not related to Azarquiel's trepidation models. See Goldstein & Chabás [1994:34-35] and [1996:325-330]. For other argumentations in favour of a calculation for the moment at which the precession is 0°, see J. Samsó [1997:107-110] and M. Comes [1991] and [1997].

5. On the description of the accession and recession and obliquity models

The first *bāb* of the second *maqāla* deals with the description of the model for the motion of the Heads of Aries and Libra in their equatorial epicycles as well as that of the Pole of the ecliptic in its polar epicycle.

II.1.[1] The description begins with an introduction in which it is stated that the group of Toledan astronomers (*al-jamāᶜa al-ṭulayṭuliyya*) reached the conclusion - based on the information available to them about ancient observations - that the difference between tropical year and sidereal year (calculated using a *zīj* based on the Indian astronomical tradition) corresponds to the precession of the stars. There follows an ancient version of the theory of trepidation, attributed to the Babylonians as in al-Bīrūnī's *Tafhīm*[101].

In the introduction to his *Book on the Fixed Stars*[102], Azarquiel attributes this theory to Hermes and his followers, and differentiates it from Theon's theory, which he takes as a combination of precession and trepidation, as did al-Battānī in his *al-Zīj al-Ṣābi'*[103]. However, in chapter 6[104], Azarquiel attributes the theory to the "Hatelesmat" and says that this was stated by Hermes in a book called *Book on the Longitude*. In chapter 4[105] he states that the opinion of the authors of the "Hind"[106] was closer to the truth than the opinion maintained by Hermes and the authors of the "Hatelesmat".

The last two lines of the page are badly damaged and impossible to read. The parts of the missing sentence that can be read are the beginning: "The

[101] On this version see J. Ragep [1993:397-398, 404].

[102] J.M. Millas [1943-1950:277].

[103] See J. Ragep [1996:271-272].

[104] J.M. Millás [1943-1950:320].

[105] J.M. Millás [1943-1950:304].

[106] See Pingree [1972].

orb advances 8°..."; and the end "... the ecliptic orb elevates and depresses approximately the same quantity in this time". The whole sentence appears in Azarquiel's *Book on the Fixed Stars* (p. 277) as follows: "The orb advances 8° **and then goes backwards the same amount and the poles of** the ecliptic orb elevate and depress 8° alternatively"[107].

It is worth stating here that Ibn al-Hā'im is using the verbs *irtafa*ᶜ*a wa-inkhafaḍa*, also used by al-Hāshimī (c.890) in his *Kitāb fī* ᶜ*Ilal al-Zījāt*. This confirms the opinion of E.S. Kennedy and D. Pingree that al-Hāshimī is referring to trepidation, although, as J. Ragep states, he cannot be referring to the model of the *Liber de Motu*[108]. Of course, al-Hāshimī, like Azarquiel and Ibn al-Hā'im, is talking about a simpler and older model in which the 8° apply not only for the trepidation motion but also for the motion of the pole of the ecliptic[109].

II.1.[2] The group of Toledan astronomers, through the differences found and the observations made, conjectured that the position of the ecliptic related to the equator was changing and that the beginning of the signs (*mabda' al-burūj*, i.e, the Head of Aries) moved around the equator, sometimes north of it and sometimes south of it, with equal motions in equal times. This, according to the author, explains the changes observed in the velocity of the precession, which affects the fixed stars and the different lengths of the tropical year, which in turn produce time differences in the passage of the Sun through the equinoxes. After working in this direction together and reaching complete agreement, the group of Toledan astronomers devised three different models, although all of them

[107] Also on pages 280 ("the fixed stars advance 8° and then retreat the same quantity ... and the same quantity corresponds to the elevation and depression of the poles of the ecliptic orb") and 321 ("The poles of the ecliptic orb move up and down 8° and the beginning of Aries advances 8° and retreat 8°"). Maybe this could be related to the fact that although neither Ibn Sinān nor al-Khāzin give any parameter for their models, a 4° radius for the path of the ecliptic pole appears quoted by the commentators of al-Ṭūsī's *Tadhkira*. See J. Ragep [1993:404].

[108] Ragep [1996:279].

[109] Al-Hāshimī [1:f.97r and p.225].

agree that only one offers a perfect fit for the conditions provided by the observations.

In the introduction to his *Book on the Fixed Stars*[110], Azarquiel confirms Ibn al-Hā'im's reference to the group of Toledan astronomers working together on this subject. In chapter 6 of the same book[111], he also affirms that the third hypothesis offers all the conditions required for the motion stated above.

II.1.[3] Therefore, the description of the model devised and accepted by the group of Toledan astronomers starts with the celestial sphere as it was some 50 years before the Hijra, coinciding, according to Ibn al-Hā'im, with the birth of the Prophet. At this time, the Head of Aries was on the equator and tropical and sidereal longitudes were equal; so the two schools, that of *al-Hind* and *al-Mumtaḥan*, coincide[112].

Ibn al-Hā'im is thus correcting Azarquiel[113], according to whom the date of the birth of the Prophet, in which $\Delta\lambda = 0°$, would be some 40 years before the Hijra, a date also maintained by al-Marrākushī[114].

Here begins the description of the model properly speaking. The radius of the equatorial epicycle will be approximately 4;8°, a rounding off of Azarquiel's third model radius (4;7,58°[115]), which appears in the tables of the *Book on the Fixed Stars* for the second accession and for the declination of the Head of Aries (δ)[116]. In fact, it is the related angle

[110] J.M. Millás [1943-1950:278].

[111] J.M. Millás [1943-1950:321].

[112] This paragraph is a summary of Azarquiel's beginning of section 2. See Millás [1943-1950:338].

[113] J.M. Millás [1943-1950:338].

[114] On the different opinions about the presumed date of the birth of the Prophet, in which $\Delta\lambda = 0°$, cf. J. Samsó [1997:108 -109] and M. Comes [1997]. According to the Toledan Tables $\Delta\lambda = 0°$ for year 604. See Mercier [1996:306-307].

[115] In Ibn al-Raqqām's *al-Zīj al-Shāmil* the value for the radius is 4;7,57°.

[116] J.M. Millás [1943-1950:336].

arcsin of 0;4,19,26°[117], a value also found in the corpus of *Book on the Fixed Stars*[118], attributed to Azarquiel by Ibn al-Hā'im in II.2.[1] (as 4;19,26,8°[119]), and used in VII.4.[1] (0;4,19,26 as r/60). This parameter is similar to the one used in the *Liber de Motu* (4;18,43°).

The model, as it was some 50 lunar years before the Hijra, is depicted in fol. 26r and Fig. 5. It consists of[120]:

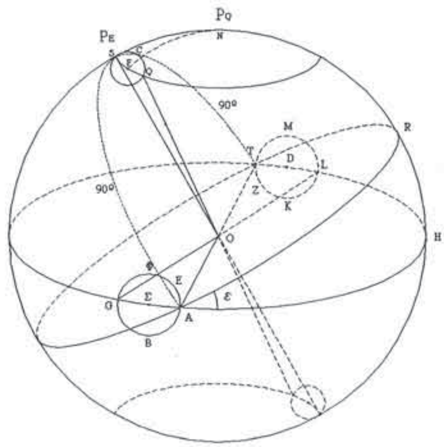


Fig. 5

ART	Ecliptic.
AHT	Equator.
O	Centre of the Universe.

[117] sin 4;7,58° = 0;4,19,26,42°, r = 4;19,26° for R = 60 in the Hebrew text edited by Millás.

[118] According to R. Mercier [1996:328], this is the value as quoted in text. J.M. Millás [1943-1950:318], states 4;19,26° on the basis of R = 60. This is also the value found in the Hebrew manuscript.

[119] Arcsin 0;4,19,26,8° = 4;7,57,27°.

[120] For the transcription of the Arabic letters I have used E.S. Kennedy [1991-1992:21-22].

A	Point at which the Head of Aries and the equator coincide.
Σ	A point at c. 4;08° from A, centre of the Aries equatorial epicycle.
AΣ	Radius of the equatorial epicycle.
ABGΦ	Aries equatorial epicycle.
TKLM	Libra equatorial epicycle.
GL & TA	Straight lines crossing at the centre of the Universe.
N	Pole of the equator.
HRNS	Solstitial colure.
HR	Angle corresponding to the obliquity of the ecliptic at that moment.
SFQ	Polar epicycle called in the text "Circle of the differences of the obliquity"[121].
<F	Centre of the polar epicycle ($r = 0;10,20°$), at 23;43° from N, which is ϵ_{mean}.
S	Pole of the ecliptic at its first ϵ_{mean}.
C	Pole of the ecliptic at ϵ_{min}.
Q	Pole of the ecliptic at its second ϵ_{mean}.
SCQ	Also called "circle of the differences of the obliquity">[122].
NS	Obliquity of the ecliptic at that moment.
SA and ST	90°
R	Summer solstice.

From this initial position, at which the Head of Aries coincides with the equator and the Pole of the ecliptic is at its medium distance from the pole of the equator (ϵ_{mean}) in direction to the minimum distance (ϵ_{min}), the motion in the equatorial epicycle will take place around the equator: first, on the North side, eastwards (with regard to the equinox), and then, on the South

[121] It seems that a number of lines are missing here, because there is no mention of the polar epicycle, nor of the value of its radius. It looks like a "saut du même à même". A suggested text appears inside angle brackets in the edition of the Arabic text.

[122] The lettering and values appearing in the following paragraphs of the text but missing in this section are shown inside angle brackets.

side, westwards (with regard to the equinox)[123]. The East is identified in the text with the place from where the *Qabūl* wind blows, and the West from where the *Dabūr* wind blows[124]. Obviously, he uses the names of the winds in connection with the names of the motions *al-iqbal* and *al-idbār*. The description of the motion coincides with Azarquiel's[125].

A similar reference can also be found in *maqala 5 bab* 1, were Ibn al-Hā'im states that when the sphere is *mudbir*, the equinox goes forward towards the East, and when the sphere is *muqbil*, the equinox goes backward towards the West[126].

II.1.[4] Let us now consider that the two equatorial epicycles move carrying the Head of Aries and Libra respectively. The Head of Aries in arc AEΦ is on the northern side of the equator and to the east of the equinoctial point, and the Head of Libra in TZK is on the southern side of the equator and to the west of the equinoctial point. The pole of the ecliptic will also have moved eastwards in the arc SC. Then the different positions change, and the situation will be the following (Fig. 6):

E	Head of Aries
Z	Head of Libra
C	Pole of the ecliptic at that moment
NC	Obliquity of the ecliptic at that moment, corresponding to ϵ_{min}
HNC	Solstitial colure
Θ	Spring equinox

[123] This interpretation is based on what the author says below in II.1.[4] and II.1.[7].

[124] On the Cardinal Winds, see M. Forcada [1994] and D.A. King [1989].

[125] See J.M. Millás [1943-1950:289-294].

[126] See E. Calvo, [1998:[29]66]. The use of "accession" (*muqbil*) and "recession" (*mudbir*) both for the equinox and the Head of Aries is also found in other sources related to Azarquiel: for instance, in the *Kitāb al-Adwār fī Tasyīr al-Anwār* by Ibn al Baqqār. Ms. Escorial, 418, fol. 237. A paragraph that seems to be Ibn al-Baqqār's source is found in Ibn Isḥāq's *Zīj*. Ms. Hyderabad Andra Pradesh State Library 298, fol. 92.

X Autumnal equinox[127]
&[128] Summer solstice at that time.

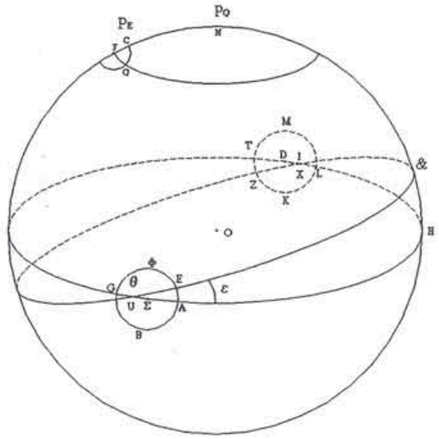


Fig. 6

II.1.[5] As the motion continues, the positions change. The Head of Aries moves in the circle AEΦ up to the point Φ, which is northern at a distance of 90° from the ecliptic; and the Head of Libra in TZK up to point K, which is southern at 90° from the ecliptic. The pole of the ecliptic will also move and arrive near the second ϵ_{mean}. The different positions will then be as follows (Fig. 7):

Φ Head of Aries
K Head of Libra
Q Pole of the ecliptic corresponding to the second ϵ_{mean}
NQ Obliquity of the ecliptic when reaching ϵ_{mean}

[127] At the equinoxes, the points Θ and I, on the ecliptic, and U and X, on the equator, mentioned here for the first time, coincide.

[128] & is used here to express the combination *lām-alif* not described by Kennedy [1991-1992].

V Spring equinox
W Autumnal equinox
J Summer solstice

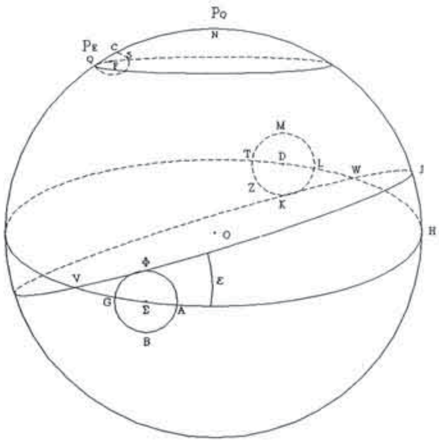


Fig. 7

II.1.[6] The motion continues in the next quarter of the equatorial epicycles (ΦG and KL), and the Heads of Aries and Libra again reach the equator at points G and L. The *al-Hind* and *al-Mumtaḥan* schools coincide again at a moment in which there is no accession nor recession, while the pole comes near its first ϵ_{mean}.

II.1.[7] The motion continues in the same way in both halves of the equatorial epicycles (GBA and LMT) as the pole rises to its ϵ_{min}. At that moment, the Head of Aries is in the southern half of the equatorial epicycle and to the west of the equinox, while the Head of Libra is in the northern half and to the east of the equinox. Exactly the opposite of the situation at the beginning of the motion.

II.1.[8] The motion in the two epicycles will be completed once the Heads of Aries and Libra have again reached points A and T.

The Heads of Aries and Libra, hence, will once again be in the position in which they were some 50 years before Hijra when the trepidation value was 0°. According, then, to Ibn al-Hā'im, the motion of the Heads of Aries and Libra takes the solstitial colure with it and this implies that there should be a connection between the two motions which justify trepidation and the variation of the obliquity of the ecliptic, for the solstitial colure passes by the different poles of the ecliptic moving around the polar epicycles. However, if these two motions have to be connected, both cannot be circular, unless the centre of the polar epicycle moves along a limited arc of the circle around the pole of the equator.

II.1.[9] The motions of the equatorial and polar pairs of epicycles each produce a pair of equal and opposed cones of a cylinder, whose bases are the circles created by these motions, connected by straight lines through the centre of the Universe. This idea comes from Azarquiel's description of his third model, although Azarquiel refers only to the cones produced by the accession and recession motion in the corresponding epicycles[129].

6. On the parameters of the motion of the Head of Aries and the Pole of the Ecliptic

Bāb 2 of *maqāla* II is entitled "On the quantity of the motions of the Head of Aries, the Pole and the Centre in their circles and the times of their return to the initial points". We will exclude the references to the centre of the solar eccentric, whose values, with the exception of the parameter of its motion for Julian years (0;6,27,42,33), are confirmed in *Maqāla* III, *bāb* 3 edited by E. Calvo in her paper on the model of the Sun in Ibn al-Hā'im's *zīj*[130], although she does not edit or comment on the references to the Sun found in the present chapter.

II.2.[1] There is no general agreement among the astronomers who established the parameters and periods of revolution. Azarquiel fixed the

[129] J.M. Millás [1943-1950:289].

[130] See E. Calvo [1998:56-58/89-90].

radius of the equatorial epicycle at 4;19,26,8°. This parameter is, according to Ibn al-Hā'im, the most accurate of those established by Azarquiel. The value, quoted by Ibn al-Hā'im, coincides with the radius stated by Azarquiel himself in his *Book on the Fixed Stars* (4;19,26°), and is even more precise[131]. As for the rest of Azarquiel's data that have reached him, Ibn al-Hā'im considers that they show a variety of imperfections.

Furthermore, Ibn al-Hā'im states the following: "When we examined the question in the pages (*ṣuḥuf*) of him (Azarquiel) that were available to us related to this question [...] we found that they disagreed with the foundations (*uṣūl*) which were the basis of our knowledge. This compels us to review the whole question in order to correct him, especially in relation to the motion of the pole. The results calculated by him concerning the values of the obliquity for the times of the observations are different from those actually observed by him and by others. The difference is not small. Therefore, we cannot build our results on this basis, not even in an approximate way, for it produces important changes in the distance between the Head of Aries and the equinoctial point".

This paragraph seems to suggest that Ibn al-Hā'im was working on the basis of some papers written in Azarquiel's own hand, whose contents may be slightly different from the ones in the *Book on the Fixed Stars*. This would explain the fact that the parameters that Ibn al-Hā'im quotes as Azarquiel's are sometimes more exact or accurate than the parameters of the *Book*. Furthermore, in Ibn al-Ha'im's *Zīj* Roser Puig has also found a reference to the fact that our author saw some writings on Azarquiel's observations of the motion of the Moon in Azarquiel's own hand[132].

II.2.[2] Ibn al-Hā'im's own values are then stated. First of all he determines the ratio (*nisba*) concerning the time elapsed between the observations of Ptolemy, al-Battānī[133] and Azarquiel, as follows:

[131] See II.1 [3].

[132] *ᶜalà mā wajadnā la-hu bi-khaṭṭ yadi-hi*. See R. Puig [2000:76-77, 91, n.[25].

[133] Jābir, in the manuscript.

Ptolemy-Battānī/Battānī-Azarquiel = $3;50,29^p/1^p$

The proportion requires a few comments: in his *Book on the Fixed Stars* Azarquiel also states this relationship, and extends it to the relationship between the observations of Hipparchus, Ptolemy and al-Battānī. According to Millás[134], the ratios are the following:

Hipparchus-Ptolemy/Ptolemy-Battānī = $1^p/2;36,37,53,40^p$

Battānī-Azarquiel/Ptolemy-Battānī = $[1]^p/[2];40,28,47,20^p$[135]

The first of Azarquiel's ratios presents no problems at all. If we adopt the dates of the "observations" of *Qalb al-Asad* calculated by Mercier[136] (Hipparchus 17/2/-145; Ptolemy 4/3/140; al-Battanī 19/3/884), we have the following ratio: $1^p/2;36,37,53,41^p$. As the fourths are stated in the text as 2/3 instead of 40, we have a very good approximation. The only correction required is the one stated by Millás for the degrees.

However, the Hebrew manuscript is confusing in relation to the second ratio[137]: to begin with, the integer parts do not appear in the text. Millás corrected the integer parts to [1] and [2], furthermore, the 40 minutes appear only in the margin of the manuscript; the seconds are corrected by Millás from 25" to 28"; the 47 thirds appear in the manuscript as fourths and the 20 fourths appear as fifths, without any mention of the thirds.

Millás's correction ($1^p/[2];[40],28,47,20^p$) would give a difference of 279 years between al-Battānī and Azarquiel, which would place Azarquiel's observations of *Qalb al-Asad* around 1165, i.e. almost one century after his time. Azarquiel uses Julian years when talking of time differences and this should also be applied to the relationships.

Considering that Azarquiel himself places his observations in 1074-1075

[134] See J.M. Millás [1943-1950:315].

[135] Neither of the integer parts are stated in the text.

[136] See R. Mercier [1996:328].

[137] I owe the readings of the Hebrew manuscript to T. Martinez.

and Mercier's calculation places them in 1076, we have to accept a correction for this parameter.

The first suggestion would be to correct Azarquiel's parameter on the basis of Ibn al-Ha'im's. If we suppose the integer parts to be 3^p instead of 2^p, and keep the 28", corrected by Millás, we would have a difference of about 203 years, meaning that Azarquiel's observations should be placed around 1087. If we also correct the 40' to 50', and forget about the fact that 47 are fourths in the manuscript, this would give us a difference between al-Battānī's time and Azarquiel's of 193 years, which places Azarquiel's observations around 1077, fitting the known data fairly well. Furthermore, $3;50,28,47,20^p$ seems to correspond to Ibn al-Hā'im's rounded off value [$3;50,29^p$].

The problem is that the aforementioned correction involves a great number of changes. There is a second possibility, which is closer to the manuscript reading. If we accept that between Ptolemy and al-Battānī the difference of time is, as stated in the above ratio, 744 Julian years, and the difference between Azarquiel and al-Battānī 196 Julian years, we have a ratio of $1^p/0;15,47,20^p$. This difference would place Azarquiel's observations around 1080. This may also be a valid reading, although it involves several changes: to apply a correction from 25" to 25'[138] and from 25' to 15'; and forget the 40' in the margin as well as the fact that 47 and 20 may be thirds, fourths or fifths, which is not clear in the manuscript.

The third possibility would be to suppose the integer parts to be 3^p and maintain the rest of the data in the manuscript. We will then have $3;40,25,0,47,20^p$ which will give us a difference of 202 years between al-Battānī and Azarquiel, placing Azarquiel's observations in 1086, exactly 10 years after the data calculated by Mercier, and 11 or 12 years after the data stated by Azarquiel himself.

It seems that the first possibility is more reasonable, mainly if we take into account the exactitude of Ibn al-Hā'im when quoting Azarquiel's words and parameters.

[138] J.M. Millás [1943-1950:315] corrects 28" to 25', although in the manuscript it appears as 25".

II.2.[3] Afterwards, Ibn al-Hā'im determines the revolution periods[139]:

a) Head of Aries: (Julian years) = 3874 y. and 3 and ca. 1/2 months.

From this revolution period, which is approximate, we can derive a daily value of 0;0,0,54,57,3,5,42° very close to the parameter given by Ibn al-Hā'im himself (0;0,0,54,57,3°) and quoted in II.2.[4]. The parameter derived from the mean motion values (0;0,0,54,57,2,41°) also fits the value stated.

The value given by Azarquiel is 3874 Julian years. As always, Ibn al-Ha'im follows Azarquiel but is more precise in the parameters.

From Azarquiel's revolution parameter, we obtain a daily value of 0;0,0,54,57,17,38,5°, very close to the parameter derived from mean motions in Persian and Julian years (0;0,0,54,57,17,38,4°). The parameter for Persian years agrees to the sevenths with the parameter for Julian years, whenever we use the value stated for 1 Persian year; if we correct this value on the basis of the values for 10 and 100 Persian years we obtain 0;0,0,54,57,7,46°. Arabic years show a different parameter, a problem which has been studied in depth by Mielgo[140].

b) Pole of the ecliptic: (Persian years) = 2032 y. and c. 29 days.

This value is once again approximate and gives a daily parameter of 0;0,1,44,50,20°, very close to the parameter given by the same Ibn al-Hā'im (0;0,1,44,49°) and also quoted in II.2.[4].

The value given by Azarquiel is 1850 Julian years. Here Ibn al-Hā'im is correcting Azarquiel. Similar corrections in the parameters of the obliquity of the ecliptic model will appear in almost all of Azarquiel's followers, because after Azarquiel's time, the obliquity was still decreasing

[139] The values are very close to those of Azarquiel and the rest of his followers, although the basic parameters seem to have been slightly modified. Azaquiel tables are calculated for Arabic, Persian and Julian Years.

[140] See H. Mielgo [1996].

and the parameters had to be modified in order to fit the observations[141]. According to Ibn Isḥāq and Ibn al-Raqqām, to make a complete revolution the pole of the ecliptic needs 2698 Julian years.

II.2.[4] From the revolution parameters he obtains the following motions:

a) Head of Aries:

(Julian year) = c. 0;5,34,30,45,40°
(Persian year) = c. 0;5,34[142],17,1,24°
(Arabic year) = 0;5,24,32,43°
(Daily) = 0;0,0,54,57,3°

The values stated by Azarquiel for the mean motion of the Head of Aries are:

(Julian year) = 0;5,34,32,16,36°
(Persian year)= 0;5,34,18,32,16,35°
(Arabic year) = 0;5,24,32,23,21,40°
(Daily) = not stated.

b) Pole of the ecliptic

(Julian year) = c. 0;10,38,4,43,3°
(Persian year) = 0;10,37,38,30,45,48°
(Arabic year) = c. 0;10,19,3,56,16,48°
(Daily) = 0;0,1,44,49°

Ibn al-Hā'im's daily motion of the Head of Aries, recalculated from Persian and Julian years, is slightly different from the daily motion

[141] See M. Comes [1996].

[142] In ms. 44.

recalculated from Arabic years. The difference starts in the sixths.

(Daily motion from Persian year) = 0;0,0,54,57,2,41,48°
(Daily motion from Julian year) = 0;0,0,54,57,2,41,45°
(Daily motion from Arabic year) = 0;0,0,54,57,2,43,33°

This also happens with the motion of the Pole of the ecliptic. The difference also starts in the sixths.

(Daily motion from Persian year) = 0;0,1,44,49,4,14,17°
(Daily motion from Julian year) = 0;0,1,44,49,4,14,6°
(Daily motion from Arabic year) = 0;0,1,44,49,4,6°

Similar differences are also found when recalculating Azarquiel's daily value for the motion of the Head of Aries[143], although in this case the difference starts in the fourths.

(Daily motion from Persian year) = 0;0,0,54,57,17,38,4,1°[144]
(Daily motion from Julian year) = 0;0,0,54,57,17,38,4,11°
(Daily motion from Arabic year) = 0;0,0,54,56,59,24,2,50°

However, in Azarquiel's motion of the Pole of the ecliptic[145], the difference does not start until the sevenths.

[143] On the differences between the daily parameter derived from Azarquiel's Arabic and Persian/Julian years, see H. Mielgo [1996:172-178]; and M. Comes [1996:357-359]. Mielgo worked on the assumption that Millás's edition was correct; though there are several misreadings, the differences do not change the main idea of Mielgo's paper.

[144] As corrected by Mielgo [1996:174-178].

[145] There is a mistake in Millás's translation of the Hebrew text. I have used the correct value which is the one that appears in the tables edited by Millás [1943-1950:329] (0;11,19,40°), different from the value stated in the translation for 1 Arabic year, p. 327 (0;10,19,59,39,40°). From the values for 10 and 100 years in the text the daily value recalculated is 0;11,19,39,59,39,40°, which corresponds exactly to the value stated in the Hebrew text, rounded off in the table to 0;11,19,40°

(Daily motion from Persian year) = 0;0,1,55,4,42,42,5,44°
(Daily motion from Julian year) = 0;0,1,55,4,42,42,5,35°
(Daily motion from Arabic year) = 0;0,1,55,4,42,42,4,17°

As we have seen, Ibn al-Hā'im's parameters do not coincide with Azarquiel's nor with the values found in the rest of Azarquiel's followers[146]. However, this is not surprising; Ibn al-Hā'im himself states that he determined new parameters on finding substantial differences amongst his predecessors. Ibn al-Hā'im is just accepting some of the old parameters, and in the case of the trepidation model he considers only Azarquiel's radius of the equatorial epicycle as correct. Ibn al-Raqqām[147] will also introduce slight corrections to Ibn al-Hā'im's parameters.

7. On the impossibility of constructing an everlasting table to determine $\Delta\lambda$.

Bāb 3 of *maqāla* II deals with the relationship between the revolution periods of the Head of Aries and the pole of the ecliptic.

II.3.[1] According to Ibn al-Hā'im, the ratio (نسبة) of the velocities is approximately the following: Rev. Pole/Rev. Aries = 1;54,40/1. Then, the revolution period of the pole of the ecliptic is almost twice the revolution period of the Head of Aries, the difference being only 0;5,20.

II.3.[2] From this difference, Ibn al-Hā'im calculates the relationship between the recurrences of both motions, using different procedures. First of all taking a revolution to be 60, he has:

11 revolutions and 1/4 for the Head of Aries.

[146] On the parameters used by the different astronomers dealing with trepidation, see M. Comes [1996:357-363]. See also J. Samsó & E. Millás [1998:262].

[147] M. Abdulrahman [1996b:51].

21[148] revolutions and 1/2 for the Pole of the ecliptic.

In effect, 60/5;20 = 11;15 and 11;15 × 1;54,40 = 21;30.

Then he uses the term "bsṭ"[149], which means that he reduces the aforementioned values to a common denominator, obtaining:

45 revolutions for the Head of Aries.
86 revolutions for the Pole of the ecliptic.

We can see that 11 1/4 = [(11×4)+1]/4 = 45/4 and 21 1/2 = [(21×2)+1]/2 = 43/2. Then, multiplying both fractions by 4, we have 45 and 86, which are the minimum value of entire numbers which keep the same ratio as that of the two velocities.

He then repeats the operation giving the revolution the value of 360° instead of 60^p. Therefore the ratio 5;20/60 becomes 5;20/60 × 360° = 32°, which means that the pole of the ecliptic will miss completing two revolutions by 32° (360° × 2 - 32° = 688°), while the Head of Aries performs one revolution.

Furthermore, 360° / 32 = 11;15° and he repeats here that 11;15 revolutions of the Head of Aries would correspond to 21;30 revolutions of the Pole.

He then uses again the term "bsṭ", meaning to reduce an entire number and a fraction to a common denominator, and he obtains:

180 revolutions for the Head of Aries
344 revolutions for the Pole of the ecliptic

In effect, 1;54,40 = 1 + 54/60 + 40/3600 = 6880/3600 = 344/180. However, 180 and 344 are not the minimum entire numbers in the aforementioned ratio of velocities, although if we simplify the ratio

[148] 22 in the manuscript: this is obviously incorrect because the ratio would be more than double; furthermore, 21 is explicitly stated at the end of this very same chapter.

[149] Ibn al-H'im uses "bsṭ", which, according to Souissi [1968:90-92], in a mathematical context means to reduce an entire number and a fraction to a common denominator.

180/344, we obtain 45/86 as before. Of course, the 344 revolutions can also be deduced from the previous step: 360 - 32/2 or 688/2.

This is the proof he needed to demonstrate that it is impossible to construct an everlasting table to determine $\Delta\lambda$. The reason is that the table will only again be useful when the Head of Aries has completed the 45 revolutions, which, bearing in mind that the Head of Aries completes a revolution every 3874 years, gives us 174330 years, a period too long to be considered.

II.3.[3] According to him, Azarquiel, Abū Marwān and the *Qāḍī* Abū 'l-Qāsim Ṣāᶜid understood this impossibility and this is why there are no tables of this kind in their works.

However, the trepidation tables in the *Liber de Motu* and in the *Toledan Tables* give directly the precession value. This suggests two possibilities: that neither Abū Marwān nor Ṣāᶜid or Azarquiel were the authors of the *Toledan Tables* or that the trepidation tables were a later addition. This latter option, however, conflicts with Mercier's proved opinion that these tables were designed to work only with the *Toledan Tables*[150].

II.3.[4] Ibn al-Hā'im states that more recent authors, amongst them Ibn al-Kammād and some contemporaries of Ibn al-Hā'im, were unaware of this and mistakenly prepared everlasting tables for $\Delta\lambda$[151] and believed that the two motions coincide[152].

8. On how to determine ϵ using the corresponding tables

Bāb 1 of *maqāla* III is devoted to determining the obliquity of the ecliptic using the corresponding tables.

[150] Mercier [1996:299].

[151] Ibn al-Hā'im himself prepared tables for the motion of the Head of Aries and the Pole of the ecliptic, but to be used for a limited period of time. See 0.[13].

[152] See 0.[6-7].

III.1.[1] It involves the use of two tables: 1) a mean motion table for the motion of the Pole of the ecliptic around its epicycle, then giving angle *j*; and 2) a table for the different values of obliquity in which the argument is *j*. The procedure suggests the use of seconds, which do not appear in all the obliquity tables -al-Marrākushī's[153] for instance- and corresponds exactly with Azarquiel's model.

9. On how to determine the $\Delta\lambda$ using the corresponding tables

Bāb 2 of *maqala* III deals with the use of the tables to determine the distance between the Head of Aries and the spring equinox, that is the accession and recession motion.

III.2.[1] Ibn al-Hā'im uses two tables (See Fig. 9):
a) a table for the mean motion of the Head of Aries (angle *i*).
b) a table giving directly $\Delta\lambda$, using angle *i* as argument.

The first table calculates the motion of the Head of Aries in its epicycle, while the second one computes the distance between the moving Head of Aries and the equinox, which corresponds to the increase or decrease of the longitude due to accession and recession motion. The second table is calculated to the precision of seconds. The result should be added to or subtracted from the sidereal longitude of the star or planet, depending on whether the equation is positive (*iqbāl*) or negative (*idbār*).

Although he is describing Azarquiel's third model, he does not use the tables and the procedure given by Azarquiel but the ones found in the *Liber de Motu* and the *Toledan Tables*, and used by most of Azarquiel's followers such as Ibn al-Kammād, Ibn Isḥāq al-Tūnisī, Ibn al-Bannā', Abū 'l-Ḥasan al-Murrākushī, Abū 'l-Ḥasan al-Qusanṭīnī, Ibn ᶜAzzūz al-Qusanṭīnī, Ibn al-Raqqām and the authors of the *Barcelona Tables*[154]. The main difference is that Azarquiel does not have a table giving $\Delta\lambda$

[153] See M. Comes [1996:362-3].

[154] See at this respect M. Comes [1996:356-362].

directly, but uses an indirect procedure involving a number of calculations[155].

As we have seen, the use of a table that gives $\Delta\lambda$ directly seems to be precisely what he has been criticizing in Ibn al-Kammād and some of the astronomers of his own time[156]. Howeveas I understand it, what he is criticizing is not the use of the table for a short period of time whenever great accuracy is not required, but the use of an everlasting table.

III.2.[2] In fact, he is merely explaining how to use a table of this kind but warning the user that when the mean motion of the Head of Aries is greater than 6 signs the approximation obtained is worse.

We have to take into account that the tables are calculated using a fixed obliquity, while the calculation with the formula allows the use of the obliquity corresponding to any moment.

In *maqāla* 7, *bābs* 3 and 4 he will explain how to determine the $\Delta\lambda$ exactly.

As we will see, in the corresponding commentary[157], he will use the same procedure and trigonometrical formula as Azarquiel.

10. On how to determine the minimum obliquity from an observed obliquity

In *maqāla* VII, *bāb* 1, Ibn al-Hā'im uses a procedure of spheric trigonometry to determine ϵ_{min} from a given observation of the obliquity of the ecliptic.

The model for the obliquity of the ecliptic described by Ibn al-Hā'im is Azarquiel's. It is depicted in fol. 81r[158], partially erased by moisture; it is also shown in Fig. 3, and described in VII.1.[4].

[155] See J. Samsó [1994:22-25].

[156] See E. Calvo [1997] and J.L. Mancha [1998:6].

[157] See VII.3. [1-4].

[158] In the figure appearing in the manuscript, letters B and G are interchanged. I have corrected this mistake following the description of the model found in VII.1.[4].

VII.1.[1] Consists of a set of instructions for obtaining ZA. Following Azaquiel, Ibn al-Hā'im uses here the cosine theorem formulated in al-Andalus by Ibn Muᶜādh in his *Kitāb Majhūlāt Qisī al-Kura*[159] and by Jābir b. Aflaḥ in his *Islāḥ al-Majisṭī*[160].

First, Ibn al-Hā'im obtains ZD, using the angle of rotation around the polar epicycle at the moment of the observation, the radius of the polar epicycle, and the observed obliquity. The procedure is as follows:

$$\text{Sin BG} = 60 \times \sin \text{BG};$$
$$0;0,10,20^\circ \times \text{Sin BG} = 0;10,20^\circ \times \sin \text{BG}$$

In order to obtain the arc of great circle GD, he operates:

$$\text{GD} = \text{Sin}^{-1}\,(0;10,20^\circ \times \sin \text{BG})$$

He then obtains Cos GD, which will be a divisor (*imām*) in the final formula:

$$\text{Cos ZD} = \text{Cos ZG} \times 60 \,/\, \text{Cos GD}$$

At this point in the text there seems to be a mistake or a copyist's error, for it states that we should obtain the arcsine (instead of the arccosine) of ZD.

Then, Ibn al-Hā'im gives the instructions for obtaining AD, again using the motion of the pole (Cord j for $j < 180^\circ$) or (Cord $(360^\circ - j)$ for $180^\circ < j < 360^\circ$) and the radius of the polar epicycle.

$$\text{Cord AG} = 60 \times \text{cord AG}$$
$$0;0,10,20^\circ \times \text{Cord AG} = 0;10,20^\circ \text{ cord AG}$$

In order to obtain the arc of great circle AG, he operates:

[159] On Ibn Muᶜādh's trigonometry see M.V. Villuendas [1979] and J. Samsó [1980:60-68].

[160] See R. Lorch [1975:38] and J. Samsó [1980:64].

$$AG = Cord^{-1} (0;10,20^{\circ} \times cord\ AG)$$

He then obtains Cos AG, which will be used in the final formula:

$$Cos\ AD = Cos\ AG \times 60 / Cos\ GD$$

As above, the arcsine of AD is used instead of the arccosine.

Finally he obtains ZA, which is the desired minimum obliquity, that is to say the minimum distance between the pole of the ecliptic and the pole of the equator, through a simple subtraction (ZA = ZD - AD).

The radius given by Ibn al-Hā'im is 0;10,20° (0;0,10,20 × 60[161]) which does not correspond exactly to Azarquiel's r = 0;10°. However, as we have seen in II.2.[1], Ibn al-Hā'im does not consider Azarquiel's radius of the polar epicycle as one of the correct values, so it is not strange to find it corrected.

VII.1.[2] By the usual formula *wa-l-ᶜilla fī dhālika*, he introduces the description of the figure as follows:

AGB	Polar epicycle
BAZ	Arc of a great circle passing through the centre of the polar epicycle, the pole of the equator and point A
Z	Pole of the equator
A	Pole of the ecliptic at its minimum distance[162] from the pole of the equator (AZ = ϵ_{min})
B	Pole of the ecliptic at its maximum distance from the pole of the equator (BZ = ϵ_{max})
G	Pole of the ecliptic at a given moment
ZG	Arc of a great circle from G to the pole of the equator (Z), that is the distance between the two poles at the given moment = observed ϵ.

[161] Both Azarquiel and Ibn al-Hā'im insist repeatedly that they use R=60. See VII.4.[1].

[162] The text states merely "distance from the pole of the equator".

AZ Minimum obliquity of the ecliptic (ϵ_{min})

VII.1.[3] Furthermore, he determines that if we know an observed obliquity, the desired minimum obliquity, that is ZA, is also known. He describes the triangles he needs to solve and adds the following:

AG Arc of a great circle, represented in the drawing by the chord AG
GD Arc of a great circle perpendicular from point G to the arc of great circle AB, represented in the drawing as a stright line

VII.1.[4] The known data are the following: BG (derived from AG); Sin BG = DG; ZG (known by observation); R = 60^p and r (radius of the polar epicycle = $0;10,20^p$).

VII.1.[5] He shows here how to solve the right angled triangle ZGD, made of arcs of great circles, in which D is the right angle. He uses the cosine theorem to determine ZD: cos GZ/cosZD = cos GD/sin 90° (the formula used in VII.1.[1]).

VII.1.[6] He then solves the right angled triangle AGD, using the known data, in order to determine AD. The procedure used here, as elsewhere, is extremely careful: arc AG of the equatorial epicycle is known (for r = 0;,10,20), we can thus calculate chord AG (for R = 60) and obtain, from a table of chords, the arc AG of a great circle of the sphere.

VII.1.[7] From these known data, he again uses the cosine theorem to determine AD: cos AG/cos GD = cos AD/sin 90° (the formula used in VII.1.[1]). Then, he only needs to subtract AD from ZD to obtain ZA, the desired minimum obliquity.

11. On how to determine the obliquity (ϵ) corresponding to a given moment

Maqāla VII, *bāb* 2, shows how to determine the obliquity of the ecliptic for a given moment.

VII.2.[1]-[4] The procedure is the same as before but in the reverse order. First of all he uses the cosine theorem to solve the right angled triangle AGD.

The known data are as before: AG, and consequently BG, which implies that DG, being the sine of BG, is also known. With this he determines DA, using the same procedure as in VII.1.[1],[6].

Afterwards, he uses right angled triangle DZG. Here, the difference is that instead of knowing the hypotenuse and one side and then determining the other side he knows the two sides and has to determine the hypotenuse.

The known data are: ZD, which is ZA (ϵ_{min}) plus DA, the value above determined and DG, being the sine of BG.

To determine ZG, that is, the hypotenuse, he uses the relationship mentioned in VII.1.[5], based on the cosine theorem. ZG will correspond to the desired obliquity for a given moment.

This is exactly what Azarquiel does in his *Book on the Fixed Stars*[163], where he also solves the same two right angled spherical triangles[164] to determine the obliquity of the ecliptic, using as known data the maximum and minimum obliquity and determining the obliquity for a given moment. He uses the same theorems as Ibn al-Hā'im, but more than a century before, more or less at the same time as or slightly after Ibn Mu^cādh, without specifying the procedure and the relations as Ibn al-Hā'im does. Azarquiel is just showing that in right angled spherical triangles, once one of the sides and the hypotenuse are known the other side is known; and once the two sides are known the hypotenuse is known, which implies the use of spherical trigonometry as in Ibn al-Hā'im's text.

12. On the knowledge of "al-iqbāl al-awwal"

Bāb 3 of *maqāla* VII deals with the determination of the *iqbāl al-awwal*, which computes $\Delta\lambda$, that is the positive or negative amount of precession

[163] J.M. Millas [1943-1950:330].

[164] According to Ibn al-Hā'im "*min qisīy dawā'ir 'iẓām*" and following Millás' translation of Azarquiel's Hebrew text "Arcos de circulo grande" (Arcs of great circles).

for a given moment, which is not found tabulated in Azarquiel. The title introduces for the first time the notion of accession and recession "perceptible by the senses" (*maḥsūs*) in contrast with the title of *bāb* 4, dealing with *al-iqbāl al-thānī*, in which the accession and recession is "perceptible by the intellect" (*maᶜqūl*).

Azarquiel also refers to *al-iqbāl al-awwal* as perceptible by the senses; however, he considers *al-iqbāl al-thānī* as the "true" *iqbāl wa-idbār* of the Head of Aries, that is the motion of the Head of Aries not in its equatorial epicycle, and hence in the ecliptic, but projected onto the equator.

The astronomers who follow Azarquiel's trepidation model consider that the outermost orb (*al-aqṣā*) has two motions, one tropical and perceptible by the senses (*ṭabīᶜī ḥissī*) and the other sidereal and perceptible by the intellect (*dhātī ᶜaqlī*)[165].

VII.3.[1] This paragraph corresponds almost verbatim to a paragraph in chapter 8 of Azarquiel's *Book on the Fixed Stars*[166] and explains the use of the table for the declination of the Head of Aries.

Should we want to calculate the accession or recession for a given moment, we will take as argument the mean motion of the Head of Aries and use it to enter the table of "declinations" in Azarquiel, or "sine of the declination" for Ibn al-Hā'im[167]. If the argument is between 1° and 180°, δ will be northern; on the other hand, between 180° and 360°, it will be southern. Then, we should apply the formula:

$$\sin \Delta\lambda = \sin \delta \times 60 / \sin \epsilon$$

If i is between 1° and 90° or between 270° and 360°, the Head of Aries is moving towards (*muqbil*) the north, while in the rest of the epicycle it is

[165] This terminology is found in Ibn al-Raqqām's *al-Zīj al-Shāmil fī Tahdhīb al-Kāmil*, chapters 72 and 73, taken from Ibn al-Hā'im, but also in his *al-Zīj al-Mustawfī* (chapter 17) and other sources such as Ibn Isḥāq's *zīj* (Hyderabad, fol. 92) and Ibn al-Baqqār's *Kitāb al-Adwār* (fol. 139v).

[166] See J.M. Millás [1943-1950:335-337].

[167] See in this regard 0.[12].

moving towards (*muqbil*) the south. If the declination is northern, the accession and recession (*iqbāl* and *idbār*) -one should understand the position of the Head of Aries- are to the east of the spring equinox and the Head of Aries will be forward (*mutaqaddima*) towards the East, but if the declination is southern, the accession and recession (*iqbāl* and *idbār*) will be to the west of the equinox while the Head of Aries will be backward (*muta'akhkhira*) towards the West[168].

If we follow Azarquiel's and Ibn al-Hā'im's instructions for calculating $\Delta\lambda$ step by step, the only difference we find is that Azarquiel uses one table for the declination and another one for sines, while Ibn al-Hā'im's table, as we have seen, gives the sine of the declination directly.

VII.3.[2] Introduced as is customary by the formula *wa-l-ᶜilla fī dhālika*, this section presents the trigonometrical explanation. To begin with, we find the description of the model, which corresponds to Azarquiel's geometrical description of chapter 8[169]. The lettering in the figure that appears in the manuscript (81v) does not correspond to the description of the text. I have reproduced it as Fig. 8, although I have followed the lettering in the text:

ABG	Equatorial epicycle
AG	Diameter of ABG
B	Head of Aries at a given moment
BZ	Arc of a great circle perpendicular to AG and δ at that moment
DB	Section of the ecliptic and $\Delta\lambda$ perceptible by the senses at that moment

[168] The possible confusion arises from two facts: the first is that place were the "accession or recession" (in Azarquiel's terminology) of the Heads of Aries and Libra starts ($i = 0°$ or $180°$) is not the place were the "accession or recession" of the equinoctial points starts ($i = 90°$ or $270°$), and the second is that neither Azarquiel nor Ibn al-Hā'im sate explicitly whether they are talking about the direction of the Heads of Aries and Libra, their positions with regard to the equinoxes or the forward and backward motion of the equinoxial points.

[169] Millás [1943-1950:333-334].

AGD Section of the equator

D Intersection between equator and ecliptic and hence the spring equinox at that moment

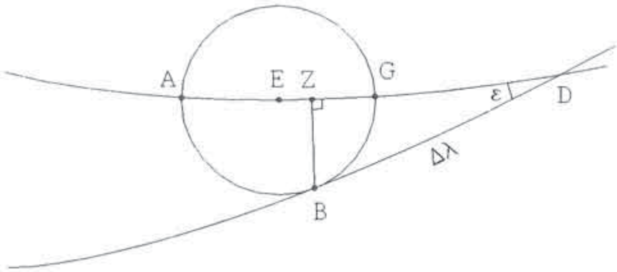


Fig. 8

This implies one of the following possibilities: either 1) Azarquiel knew Ibn Muᶜādh's trigonometry, which apparently was unknown in Toledo in the time of Ṣāᶜid al-Andalusī or 2) The new Eastern trigonometry reached al-Andalus not only through Ibn Muᶜādh but via other channels as well.

VII.3.[3] Ibn al-Hā'im then determines the relationship for solving the spherical triangle DZB (Fig. 8), which does not appear in Azarquiel, although it is implicit in his geometrical resolution of the spherical triangle. Ibn al-Hā'im uses the sine theorem here, formulated also in al-Andalus by Ibn Muᶜādh in his *Kitāb Majhūlāt Qisī al-Kura*[170] and Jābir b. Aflaḥ in his *Īslāḥ al-Majisṭī*[171].

$$\text{Sin } BZ/\text{Sin } BD = \text{Sin } D/\text{Sin } Z$$

[170] On Ibn Muᶜādh's trigonometry see M.V. Villuendas [1979] and J. Samsó [1980:60-68].

[171] See R. Lorch [1973:38] and J. Samsó [1980:64].

VII.3.[4] Ibn al-Hā'im tries to determine BD = $\Delta\lambda$, from the known data, that is: BZ = δ; D = ϵ; Z = 90°, of a right angled spherical triangle. Then, according to the given relation, the formula to be applied has to be:

$$\sin \Delta\lambda = \sin \delta \times \sin 90° / \sin \epsilon.$$

We find exactly the same in Azarquiel's text.

13. On the knowledge of the "iqbāl al-thānī"

As we have seen, *bāb* 4 of *maqāla* VII deals with the determination of the *iqbāl al-thānī*[172], which calculates what Azarquiel calls the "equation of the diameter" (ZG in Fig. 9). As usual, the figure in the text (fol. 82v) is incomplete. The *iqbāl al-thānī* is the difference between the right ascensions of the Head of Aries (B) and of point (G). In Ibn al-Hā'im's words *iqbāl al-ʿamūd fī 'l-quṭr*.

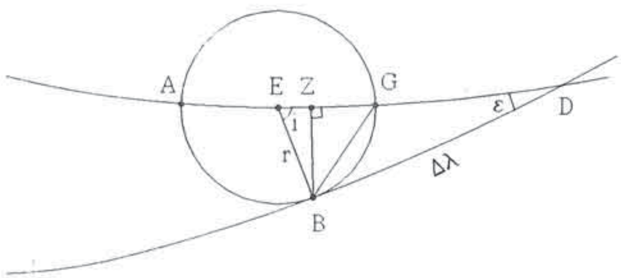


Fig. 9

[172] On *al-iqbal al-thānī* see B. Goldstein [1964:242-244]; and J. Samsó [1994b:X;15].

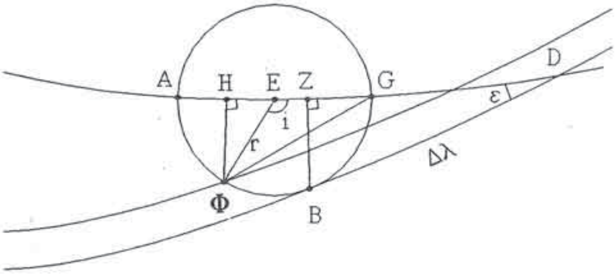


Fig. 10

The terminology used needs some commentary. The *iqbāl al-thānī* is an arc on the equator, while the *iqbāl al-awwal* is an arc on the ecliptic. The *iqbāl al-thānī* is defined in the title of the chapter as if it were the ascension of each degree of the equatorial epicycle with respect to the meridian. The same kind of expression appears again in [3] and is clarified in [4], in which the words *mā yaṭla^c fī dā'irat nisf al-nahār* become *mā yajūz fī dā'irat nisf al-nahār*. This seems to refer to the arc of the equator between the meridian transits of two points of the equatorial epicycle.

In *bāb* 17 of *maqāla* VI, Ibn al-Hā'im uses the *iqbāl al-thānī* to calculate the equation of time. The equation of time is the difference between two passages of the sun through the meridian, the passage of the real sun traveling along the ecliptic and the hypothetical passage of the mean sun traveling along the equator. He uses the *iqbal al-thānī* to determine the precession value projected onto the equator, that is the right ascension of $\Delta\lambda$[173]. The same use is mentioned in chapters 6 and 17 of Ibn al-Bannā's *Kitāb Minhāj al-Ṭālib li-ta^cdīl al-Kawākib*[174].

[173] On this, see A. Mestres [1999:I,29-31]

[174] See Vernet [1952:28,82;53,117].

VII.4. [1-2] Paragraph one partially corresponds to a paragraph in chapter 8 of Azaquiel's *Book on the Fixed Stars*[175]. In it we find the instruction for calculating *al-iqbāl al-thānī* with the corresponding table using the mean motion of the Head of Aries as argument.

However, Ibn al-Hā'im here presents a formula not found in Azarquiel to calculate the *al-iqbāl al-thānī* for the positions of the Head of Aries taking into account if $i < 180°$ or $i > 180°$.

In the first case, he uses:

Cord $i \times 0;4,19,26^p$ = 60 cord $i \times 0;4,19,26^p$ = cord $i \times 4;19,26^p$

Sin $(180 - i) \times 0;4,19,26^p$ = 60 sin $(180 - i) \times 0;4,19,26^p$ =

sin $(180 - i) \times 4;19,26^p$

The development of the formula is as follows:

Cord^{-1} (cord $i \times 4;19,26^p$) = ⌒BG (arc of great circle)

Sin^{-1} (sin (180 - $i \times 4;19,26^p$) = ⌒BZ (arc of great circle)

He, then, determines ZG, the desired *iqbāl al-thānī*, using the relationship stated in VII.4.[4], where we find the geometrical explanation, corresponding to Azarquiel's[176]:

Cos ZG = Cos BG × 60 / Cos BZ;

For the second case, the procedure is the same, using:

[175] J.M. Millás [1943-1950:336-337].

[176] J.M. Millás [1943-1950:333-334].

Cord (360 - *i*);

Sin (180 -(360 -*i*)) = Sin (*i* - 180)

The radius of the equatorial epicycle is 4;19,26^p (0;4,19,26^p × 60). As in the case of the radius of the polar epicycle, Ibn al-Hā'im uses two values, the radius given in the text and the value of the radius divided by 60, used in the geometrical description.

The description found in VII.3.[2] is also repeated, with the following additions:

E Centre of the epicycle
BG Arc of great circle and straight line connecting the two points

VII.4.[3] Now, the known data are: GB (*i*); BA (180° - *i*) and ZB (δ).

The geometrical explanation is as follows: given the triangle GBZ, made of arcs of great circles, of which Z is the right angle, the relationship, according to Ibn al-Hā'im but not stated by Azarquiel, will be:

$$\cos BG / \cos GZ = \cos BZ / \sin 90°$$

As BG and BZ are known, we also know GZ, that is, *al-iqbāl al-thānī*. So Ibn al-Hā'im is using the cosine theorem here. Azarquiel uses the same triangle and the same data so that, implicitly, he is using the same relationship.

14. On the knowledge of the "iqbāl al-thānī" between two given positions of the Head of Aries

VII.4.[4] Then, Ibn al-Hā'im adds the procedure to be used to determine the "second accession" between two different positions of the Head of Aries. Let us now suppose that at another moment the Head of Aries is at point Φ and draw a perpendicular to AG, in order to obtain the value of δ at that moment. We also draw an arc of a great circle between Φ and G and join the two points with the straight line ΦG (Figs. fol. 83v. and 10).

VII.4.[5] Then, let us consider, as before, the right angled triangle ΦHG made of arcs of great circles, in which we know: H = right angle; HΦ (δ) and ΦG (i). We will calculate GH using the cosine theorem. As GZ is already known (VII.4.[3], once GH is determined, HZ will also be known and this is the quantity corresponding to the second accession between the two positions of the Head of Aries corresponding to two given moments.

III. Conclusions

1. General conclusions

This manuscript is interesting because it offers, for the first time, a detailed description of the Andalusian theory of trepidation in an Arabic text and not in a Latin or Hebrew translation of an Arabic text. In the *Kāmil zīj*, accession and recession, or trepidation, is a constant throughout the book because it has to be taken into account for most of the calculations and procedures that the book presents: from the equation of time and the explanation of the different kinds of years to the knowledge of the procedure used to calculate the tropical longitudes of the heavenly bodies referred to the ecliptic.

Ibn al-Hā'im is not keen on tables; however, he suggests that they may be used, whenever the user is aware of the fact that, in general, tables have a time limit of no more than 40 years. In particular, trepidation tables have to be adapted to the changing obliquity. In fact Ibn al-Hā'im's equation table takes into account the value of ϵ for each mean position of the Head of Aries on the equatorial epicycle (See 0.13). However, the fact that the period of revolution of the Pole of the ecliptic is not exactly twice the period of revolution of the Head of Aries implies that the values for arguments between 180° and 360° in the table will not be symmetrical to those between 0° and 180°. Even if the table is prepared for 360°, it will be valid just for one revolution of the Head of Aries.

Ibn al-Hā'im also offers other information on Andalusian sources: for instance, the parameters used by Abū Marwān al-Istijjī, which suggest that he wrote a *zīj* other than the *Toledan Tables*. In fact, Abū Marwān's parameters differ from the known parameters, although his determination of the increase of precession between Hipparchus and Ptolemy coincides with Azarquiel's. Furthermore, his parameters seem not to be related to those of the *Toledan Tables* or the *Liber de Motu*. Ibn al-Hā'im also introduces us a certain *Muntakhab zīj*, by a contemporary of his, whose values for precession corresponding to Hipparchus and Ptolemy's time are very close to the values in Azarquiel's second model. And the table for the increase or decrease of precession in this *zīj* seems to have coincided with the one in Ibn al-Raqqām's *al-zīj al-Qawīm* and Abū 'l-Ḥasan al-Qusanṭinī's *zīj*, which differ from the tables found in the rest of Azarquiel's followers.

To determine Ibn al-Hā'im's role in the spreading of the trepidation models, I will analyze his relationship with his forerunners, Azarquiel (III.2) and Ibn al-Kammād (III.3), and with his followers, the author of the Hyderabad recension of Ibn Isḥāq's *zīj* (III.4) and Ibn al-Raqqām (III.5).

2. Ibn al-Hā'im and Azarquiel

As we have seen, in the *Kāmil zīj*, Ibn al-Hā'im relies heavily on the models in Azarquiel's *Book on the Fixed Stars*. He uses Azarquiel's third trepidation model as well as his model for the obliquity of the ecliptic. Although it has always been considered that in Azarquiel's third model the obliquity of the ecliptic model was independent of the trepidation model, Ibn al-Hā'im, following Azarquiel, connects the two motions and models. In them, the pole of the ecliptic carries the solstitial colure and maintains a constant distance of 90° from the Heads of Aries and Libra.

Ibn al-Ha'im is, however, more precise in the parameters given, perhaps indicating that he is not using the *Book on the Fixed Stars*, but some sheets of paper written in Azarquiel's own hand; in fact, Ibn al-Hā'im states this in a couple of places of this book.

As far as the geometrical resolution is concerned, both used spherical trigonometry. In the *Kāmil zīj*, Ibn al-Hā'im adds to Azarquiel's text the detailed description and use of spherical trigonometry, as regards to formulae and relationships between sides and angles of right angled spherical triangles, showing a good knowledge of the subject (See VII.1 and VII.2)

As far as the use of the tables is concerned, as we have seen, there are two basic differences between Ibn al-Hā'im and Azarquiel: 1) Ibn al-Ha'im, like all Azarquiel's followers, devises a table that gives the increase or decrease of trepidation directly. Nevertheless, he recommends Azarquiel's procedure for calculating this value due to the fact that standard trepidation tables, based on a fixed value for the obliquity of the ecliptic, are only valid for a short period of time, due to which, as we have seen, he calculates a table taking into account the different values of the obliquity corresponding to every position of the Head of Aries; 2) instead of Azarquiel's table of declinations of the Head of Aries, Ibn al-Ha'im gives a table for the "sine of the declination", because in fact this is the value needed to apply the formula $\sin \Delta\lambda = \sin \delta / \sin \epsilon$.

Furthermore, we should note that some paragraphs of Ibn al-Hā'im's text are excerpts, often word for word, from Azarquiel's book. Examples can be found in II.1.[1,3,9]; VII.3 [1,2] and VII.4.[1]. However, Ibn al-Hā'im often modifies the parameters to agree with the observations.

3. Ibn al-Hā'im and Ibn al-Kammād

Ibn al-Hā'im considered Ibn al-Kammād an ignorant disciple of Azarquiel who had distorted his master's theory. He states that Ibn al-Kammād's trepidation model is based on two fundamental errors: 1) Ibn al-Kammād believes that the motion of the Head of Aries and the motion of the Pole of the ecliptic are the same and, therefore, devises a single table for both motions; 2) he thinks that this table is valid for any time.

According to Ibn al-Hā'im, the ratio between the two motions is approximately 1/2[177] and Ibn al-Kammād's table cannot be valid for both motions or for any time, because of the obliquity of the ecliptic used in the formulae underlying the table changes.

To this it must be added that Ibn al-Hā'im states in different parts of the

[177] 1/1;54,40° in II.3.[1].

book that *zījes* have a limited duration of no more than 40 years[178]. One of Ibn al-Kammād's errors was precisely to consider his *zīj* as everlasting (*al-Amad ʿalā al-abad*). Surprisingly enough, in the Hyderabad's version of Ibn Isḥāq's *zīj*, we can read that in *al-Kawr ʿalā al-Dawr*[179] Ibn al-Kammād stated that the duration of the *zījes* was limited.

4. Ibn al-Hā'im and the Hyderabad version of Ibn Isḥāq al-Tūnisī's "zīj"[180]

As the author of this version of Ibn Isḥāq's *zīj* mentions Ibn al-Hā'im as one of his sources, which also include Azarquiel and Ibn al-Kammād, it seemed worthwhile to study the trepidation chapters in this *zīj*.

It was, however, disappointing to find that in the theoretical discussion of trepidation, the author relies only on Ibn al-Kammād's *al-Kawr ʿalā al-Dawr*.

In his tables, the author follows Azarquiel, directly or through Ibn al-Kammād, although he introduces corrects slight corrections in some of the parameters. For instance, in the daily parameter for the motion of the Head of Aries in its equatorial epycicle, Ibn Isḥāq as well as Ibn al-Bannā', Ibn al-Raqqām and Ibn ʿAzzūz al-Qusunṭīnī follow Azarquiel's value for Persian and Julian years, although adapted to Arabic years, while Ibn al-Kammād, al-Marrākushī and Ibn al-Hā'im follow Azarquiel's table for

[178] In fact, in different parts of the book Ibn al-Ha'im affirms that his tables will be useless after a period of 40 years from the date of composition, which, according to him, corresponds to the beginning of 7th c. *Hijra* (13th c. AD). See for instance fols. 36r, 70v and 71v, or when he speaks of other items such as the determination of the true longitudes of the sun by means of the corresponding tables. On this see E. Calvo [1998:57]. In the Hyderabad revision of Ibn Isḥāq's *zīj*, it is stated that the 40 years limit duration of a *zīj* is due to the changing obliquity. See the Arabic edition of the text in A. Mestres [1999:II,296].

[179] F. 92. See conclusions point 4.

[180] Hyderabad Andra Pradesh State Library ms. 298. On trepidation and obliquity of the ecliptic in this MS see M. Comes [1992:147-159] and [1996:355-364]. A general account of the contents of the *zīj* is to be found in A. Mestres [1996:383-444] and an edition of the main part of the *zīj* in A. Mestres [1999].

Arabic years, which as we have seen is different from his tables for Persian and Julian years. However, he does not propose Azarquiel's method for calculating the accession or recession value for a given moment but a table giving the value directly, as do all the rest of Azarquiel's followers.

Be that as it may, in the context of trepidation, Ibn al-Hā'im is not mentioned at all. It appears that the anonymous author was following Ibn Isḥāq directly here, and as Ibn Isḥāq was a contemporary of Ibn al-Hā'im, he did not probably even know of him.

5. Ibn al-Hā'im and Ibn al-Raqqām.

As is already known, Ibn al-Raqqām has three different *zījes*: *al-Zīj al-Shāmil fī Tahdhīb al-Kāmil*, *al-Zīj al-Qawīm fī Funūn al-Taᶜdīl wa-l-Taqwīm*[181] and *al-Zīj al-Mustawfī*. These *zījes*, especially the first one, rely heavily on Ibn al-Hā'im, although the parameters do not coincide[182].

According to Ibn al-Raqqām himself, *al-Zīj al-Shāmil fī Tahdhīb al-Kāmil*, as its name indicates, was expressly intended to provide tables for Ibn al-Hā'im's *al-Zīj al-Kāmil*, and for this reason I have used it to fill some of the blanks in the manuscript.

For the arguments in favour of or against the possibility that Ibn al-Hā'im's *zīj* had its own tables[183], I would suggest that he may have devised some tables and Ibn al-Raqqām was merely trying to complete the book with the ones that were missing.

Seven chapters of this *zīj* deal with trepidation:

- *Bāb* 17, "On the quantities of the motion of the Head of Aries, the Pole and the Centre in their circles". It corresponds to Ibn al-Hā'im's paragraph II.2.[3].
- *Bāb* 25, "On the knowledge of the obliquity of the ecliptic using the tables". It corresponds, almost word for word, to Ibn al-Hā'im's III.1.
- *Bāb* 26, "On the knowledge of the distance between the Head of Aries

[181] Cf. E.S. Kennedy [1997:35-72]

[182] See Abdulrahman [1996b].

[183] See point 3 of the commentary.

and the spring equinox using the tables". It is a summary of Ibn al-Hā'im's III.2.

- *Bāb* 70, "On the knowledge of the minimum obliquity and the distance between the two poles (ecliptic and equator) from an observed obliquity". It corresponds, almost verbatim, to Ibn al-Hā'im's VII.1.[1-3]. As in the next chapter, he uses the description of the formula, but avoids the trigonometrical explanation.
- *Bāb* 71, "On the knowledge of the value of the obliquity and the distance between the two poles at any moment". It corresponds, almost word for word, to Ibn al-Hā'im's VII.2.[1]. As in the chapter above, he gives the description of the formula, but avoids the trigonometrical explanation.
- *Bāb* 72, "On the knowledge of the distance between the Head of Aries and the spring equinox at any moment, which is the first accession and recession, perceptible by the senses". It corresponds to Ibn al-Hā'im's VII.3.[1], which in its turn corresponds to a paragraph in chapter 8 of Azarquiel's *Book on the Fixed Stars*. Ibn al-Raqqām avoids the geometric description.
- *Bāb* 73, "On the knowledge of the ascension degrees in the equator of the accession and recession circle degrees, which is the second accession, perceptible by the intellect". It corresponds to Ibn al-Hā'im's VII.4.[1]. Ibn al-Raqqām here has avoided the geometrical description as well as the trigonometrical explanation.

In the *Qawīm zīj* only one chapter refers to trepidation:

- *Bāb* 9, "On the accession and recession motion". It corresponds to *bāb* 26 in Ibn al-Raqqām's *Shāmil*.

The *Mustawfī zīj* also devotes two chapters (16 and 17) to trepidation. In chapter 16, he deals with the *iqbāl al-thānī* in his treatment of the equation of time and in chapter 17, after a long and detailed description on the use of the corresponding tables, there is a short paragraph on the differences between tropical and sidereal motions and a brief mention of the use of the formula (sin) $\Delta\lambda = \sin\delta/\sin\epsilon$ to achieve the increase or decrease of precession for a given moment.

IV. Appendix

1. Edition of the sections of the text dealing with trepidation of the equinoxes and obliquity of the ecliptic.

الزيج الكامل لابن الهائم

حركة الإقبال والإدبار والميل الكلّي

المقدّمة

[١] (٣ظ) [...] إلاّ أنّ أبا العبّاس الكمّاد رحمه الله مؤلف هذين الكتابين[184] وهم في بعض مواضع منهما وخاصّة في الكور على الدور وهو ممّا يوجب الخلل فيما ينتج منهما ويلزم بمنعها حتّى أخذ الناس في لوم الأرصاد الطليطلية ودمغها ودفعها بعد ضمّها وقد عنى بهذا الكتاب طائفة من المنتحلين لهذا الباب فكلّ صرف عن هذه المواضع ولم ينح إليها لائم ولا دافع وأينما عرضت لهم انشاء لما يحاولون عن ذلك ويتناولون فهم يمرّون عليها وهم عنها معرضون.

[٢] ولقد رأيت لأحدهم مكتوبا بخطّ يده في بعض نسخ هذا الكتاب المذكور ما يدلّ على جهله بهذه الصناعة وورمه أن ينتهض منها فيما ليس له به استطاعة على طرء الجدول الذي وضعه الكمّاد لحركة الإقبال والإدبار "هذا هو الجدول الذي عمله الزرقالة" فعجبنا من جهل هذا الرجل بذلك وكون رتبته من هذا العلم هنالك كيف لا.

[٣] وأبو إسحاق رحمه الله يقول بلفظه في مقالة الكواكب الثابتة "ولم يمك‹ن›نا عمل جدول لأبعاد رأس الحمل من نقطة الاعتدال الذي هو الأول

[184] غير واضح في المخطوط (ص. ٣و) فعلى الأرجح أن يعني بهما "المقتبس" و"الكور على الدور".

المحسوس من قبل أنّ ميل فلك البروج ليس واحداً بعينه في جميع الأزمنة فتختلف لذلك الزاوية الحادثة بين فلك البروج ومعدّل النهار فتختلف لذلك الأبعاد المذكورة للقوس الواحدة من دائرة الإقبال لاكنّا نذكر وجه استخراجها بالحساب لأيّ وقت شئنا إن شاء الله"[185] هذا هو نصّ كلام أبي إسحاق رحمه الله على حركة الإقبال.

[٤] وقد عمل إنسان آخر في زماننا هذا زيجا سمّاه المنتخب جمع فيه أوساط أبي مروان الإستجّي[186] وتعاديل البتّاني وسائر ما في الكور على الدور من الأوهام التي وهم الكـ(مّـاد) فيها (...)[187].

[٥] (٤و) [...] وسنبيّن بعد ذلك إن شاء الله عند تعديد أوهام الكمّاد رحمه الله في كتابه المسمّى بالكور على الدور وكيف يمتحن فساد ما وضع في ذلك الزيج المنتخب من حركة الإقبال والإدبار وأنّ واضعه لم يتصوّر حقيقة الوضع الذي وضع لتلك الحركة ولا فهم وجه العمل في تدوين مقاديرهما ووضع جداول لها [...] (٤ظ) وننبّه عليه في موضعه من هذا الكتاب إن شاء الله تعالى [...].

[٦] (٥و) [...] ومن ذلك أنّه[188] وهم في حركة الإقبال والإدبار من جهات إحداها أنّه ذكر في صدر الكتاب حين تكلّم على هيئة هذه الحركة أنّ قطب البروج متى كان في بعد فلكه الأوسط من دائرة اختلاف الميل الكلّي كان رأس الحمل إذ ذاك على دائرة معدّل النهار و كانت نقطتا الاعتدال ورأس الحمل نقطة واحدة فيلزم عن هذا الأصل إذاً أنّ

[185] علامة الاقتباس موضوعة من قبلي.

[186] في المخطوط جاءت كلمة "الإستجيبي" فهي غير واضحة.

[187] السطر الآخير (ص. ٣ظ) غير واضح فيتكلّم عن اقتفاء أثر الكمّاد في هذا الزيج.

[188] يعني ابن الكمّاد.

حركة رأس الحمل في دائرة الإقبال شبيهة بحركة القطب في دائرة اختلافه وأنّ زماني عودتيهما واحد وقد تبيّن بالبراهين الصحيحة أنّ الأمر فيهما على خلاف ذلك.

[٧] الجهة الثانية أنّه عمل جدولاً لهذه الحركة لتسعين درجة من دائرتي الإقبال والاختلاف واعتقد عمومه لجميع الازمان ⟨بدليل⟩ أنّه وضع في اعلاه ستّة بروج وفي أسفله ستّة بروج على النحو الذي توضع عليه جداول التعاديل وذلك محال في هذه الحركة لأنّه لم يستوف القطب في تلك المدّة جميع ميوله الكلّية.

[٨] الجهة الثالثة أنّه جعل مبدأ حركة رأس الحمل في رأس الجدول من دائرة الاعتدال وقد ⟨زعم أنّ⟩ القطب إذ ذاك كان في بعده المتوسط فلو تمّم الجدول على هذه الأصل لمائة وثمانين جزءا ⟨فما⟩ استوفى القطب أيضاً في ذلك الزمان جم⟨يع⟩ ميوله الكلّية إلاّ أن يكون مبدأ تحريك رأس ⟨الحمل⟩ (٥ظ) من إحدي نهايتيه إمّا الجنوبية وإمّا الشمالية بحيث يكون القطب إذ ذاك في أحد بعديه أمّا الأبعد أو الأقرب وبهذا الوجه وحده يكون القطب قد استوفى في نصف الدورة جميع ميوله الكلّية وحينئذ يجب على أصله عموم ذلك الجدول لجميع الأزمان.

[٩] الجهة الرابعة أنّ تلك الحركة المذكورة التي وضع للإقبال والإدبار لا تصحّ بها حركة واحدة من الحركات المرصودة وذلك أنّ الأصل الذي بنأ الراصد مقادير هذه الحركة عليه إنّما هو أن يكون إقبال إبرخس تسعة أجزاء وتسعاً وعشرين دقيقة بتقريب ط كط وحصّة[189] رأس الحمل إذ ذاك مائتا جزء واثنان وتسعون جزء ونحو من ثلاث وثلاثين دقيقة ط كب لب م وإقبال بطلميوس ستّة أجزاء واثنان وأربعين دقيقة و مب

[189] في المخطوط "خاصة".

وحصّة رأس الحمل ثلاث مائة جزء وتسعة عشر جزءًا ونحو من دقيقتين ي̄ يط̄ ب̄ فإذا دخلنا بهاتين الحصّتين في جداول تعديل حركة رأس الحمل من ذلك الكتاب خرج لنا منها أمّا إقبال إبرخس فتسعة أجزاء وتسع عشرة دقيقة ط̄ يط̄ بنقصان عشر دقائق عمّا وضع أصلًا فيها وأمّا إقبال بطلميوس فستّة أجزاء وسبع وخمسون دقيقة و̄ نز̄ بزيادة خمس عشرة دقيقة على ما وضع أصلًا في ذلك فهذا ما في حركة الإقبال من الفساد والاختلال [...].

[١٠] (٨ظ) [...] ولنذكر الآن كيف يمتحن فساد الجدول الموضوع في الزيج المنتخب لحركة الإقبال وذلك أنّ أبا مروان الإستجّي[190] رحمه الله وضع أصل حركات الإقبال والإدبار على أنّه كان مقدار الإقبال والإدبار أمّا في رصد إبرخس فتسعة أجزاء وثمانيا وثلاثين دقيقة وثلثي دقيقة ط̄ لح̄ م̄ وحصّة رأس الحمل إذ ذاك تسعة بروج وثلاثا وعشرين درجة واثنين وأربعين دقيقة ط̄ كج̄ مب̄ وأمّا في رصد بطلميوس فستّة أجزاء وخمسين دقيقة وثلثي دقيقة و̄ ن̄ م̄ وحصّة رأس الحمل عشرة بروج وتسعة عشر جزءًا وثلاثة وعشرين دقيقة ي̄ يط̄ كج̄ فإذا دخلنا بهاتين الحصّتين في جدول تعديل الإقبال والإدبار الموضوع في ذلك الزيج المذكور خرج لنا منه أمّا إقبال إبرخس فتسعة أجزاء وخمسا وأربعين دقيقة ونصف دقيقة ط̄ مه̄ ل̄ بزيادة ستّ دقائق وخمسين ثانية على ما وضع أصلًا في ذلك وأمّا إقبال بطلميوس فستّة أجزاء وخمسة وخمسين دقيقة[191] وربع دقيقة و̄ نه̄ يه̄ بزيادة أربع دقائق وخمس وثلاثين ثانية على ما وضع أيضًا أصلًا في ذلك.

[190] "الاستجبي" في المخطوط.

[191] في المخطوط "وخمسين دقيقة" فقط.

[١١] ثمّ إنّه إذا امتحن بتلك الحركة المذكورة وبتعاديل البتّاني جميع المجازات الاستوائية المرصودة لم يتصحّ به ولا واحداً منها ووجد بين زماني الاستوائين المرصود والمقوّم فرق بعيد لا يمكن للراصد وهم فيه ولا تسامح فهذا ما في هذه الكتب المحدثة من الفساد والتخليط وقد نبّهنا عليه من كان يتوخّى في علمه وعمله الحقّ ويتحرّى في قضاياه الصواب والصدق.

[١٢] ولمّا فهمنا نحن من أنصار هذه الحركة ورأينا من استحالة ضبطها وتدوين مقاديرها الجزئية في جدول يعمّ جميع الأزمان وضعنا جد[ول] الجيوب لميول رأس الحمل في جميع <أجزاء> الدائرة وذكرنا وجه ⟨العمل في⟩ استخراج مقادير (٩و) الإقبال والإدبار معاً في كلّ زمان.

[١٣] ثمّ رأينا أن لا نخلي كتابنا هذا أيضاً من جدول نضع فيه من مقادير الإقبال ما يمكن وضعه لبعض ما يأتي من الأزمان فوضعنا جدولاً لمقادير الإقبال فقط يكون يستمتع به مدّة طويلة من الزمان وذلك ما دام رأس الحمل شماليا عن معدّل النهار وحصّته دون ستّة بروج وإن استعمل ذلك الجدول في نصف الدورة الثاني وذلك في حال الإدبار إذ تكون حصّة رأس الحمل من[192] ستّة بروج إلى اثني عشر برجاً صحّ العمل به ولم يكن بينه وبين المعمول على ذلك الزمان كثير اختلاف إلاّ في مقادير يسيرة من الثواني فقط فإذا كملت هذه الدورة واستأنف رأس الحمل بمشيئة الله سبحانه دورة أخرى فالأصحّ و الأصلح أن يعمل لمقادير الإقبال والإدبار جدول أخر بجيوب ميول رأس الحمل التي وضعناها لذلك في كلّ زمان [...].

[١٤] وجميع ما أثبتناه في هذا الكتاب من الحركات المستدركة على

[192] "من" ليس موجود في المخطوط.

القدماء وغيرها فإنّما عوّلنا فيه على أرصاد ابي اسحاق الزرقالة رحمه الله وعلى أصوله التي أصّل فيها [...] (٩ظ) [...] إنّ إبرخس رصد قلب الأسد فوجده في[193] كط نَ من السرطان ثمّ رصده بطلميوس بعده بنحو من مائتين وسبع وثمانين سنة[194] في[195] بَ لَ[196] ثمّ رصده البتّاني بعده بنحو من سبعمائة[197] وإحدى و أربعين سنة فوجده في[198] يدَ درجة من الأسد ثمّ رصده الفقيه أبو عبد الله بن برغوث بعد ذلك بنحو من مائة سنة وإحدى وسبعين سنة فوجده في ستّ عشرة درجة[199] من الأسد يوَ كَ.

[١٥] وهذه الحركة هي التي نسمّيها نحن حركة الإقبال وذلك أنّ موضع هذا الكوكب المذكور في أزمان هذه الأرصاد تدلّ على أنّه كانت الأفلاك إذ ذاك مقبلة [...].

المقالة الثانية

(٢٣ظ) الباب الأوّل.

في هيئة حركة رأس الحمل في دائرة الإقبال والإدبار.

[193] يَ في المخطوط.

[194] ينقص "فوجده".

[195] يَ في المخطوط.

[196] ينقص اسم البرج في المخطوط فيجب أن يكون "من الأسد".

[197] "تسعمائة" في المخطوط .

[198] يَ في المخطوط.

[199] ينقص "وعشرين دقيقة" في المخطوط.

[١] ولمّا وقفت الجماعة الطليطلية رحمة الله عليهم أجمعين على ما وقع إليهم من أرصاد القدماء ورأت ما يلزم عنها من الخلاف والاضطراب في زمان السنة وفي حركات الثوابت ثمّ رأوا مع ذلك الخلاف الذي يوجد في مجازات الشمس الاستوائية إذا قوّمت بزيج الهند شبيهاً بالخلاف الذي يوجد لحركات الثوابت حتّى أنّه ليوجد في بعض الأوقات مساويا منطبقا عليه ثمّ أنّهم وجدوا القدماء من أهل هذه النواحى من المعمور كأهل بابل وغيرها ممّن عفت أثارهم ورسومهم ودرست أخبارهم وعلومهم من أشاروا لها ⟨...⟩هم من ذلك وبقي لهـ⟨م ...⟩ بغض هذا ⟨العمل⟩ إلى أنّ الفلك يقبل ثمانية أجزاء ⟨ويدبر شبيهاً وقطب⟩ (٢٤و) فلك البروج يرتفع وينخفض نحوا من ذلك المقدار في ذلك الزمان.

[٢] ⟨وحدسوا⟩ من هذه الأمور كلّها بحصافة عقولهم وذكاء فطرهم على أن وضع فلك البروج من معدّل النهار ليس واحداً بعينه في جميع الأزمنة ولا ثابتاً على حال واحدة وأنّ مبدأ البروج فيه متحرّك حول معدّل النهار شمالاً عنه وجنوباً حركة معتدلة سواء في جميع الأزمان وأنّ بهذا فقط يمكن أن تكون حركة الثوابت على النحو الذي وجدت عليه بالأرصاد من السرعة والبطء قديماً وحديثاً وعن ذلك يلزم اختلاف عودة الشمس إلى معدّل النهار وتوجد أزمان الاعتدالات تختلف أبداً نحوا من ذلك الاختلاف الموجود بالأرصاد ولمّا قويت ظنونهم واتّفقت عليه آراؤهم وتسددت نحوه أغراضهم ونالت إليه إهداءهم شرعوا في تصوّر هذه الحركة وشكلها فتصوّروها أنحاء من التصوّر ثلاثة إلى أنّ أحدها فقط هو الذي يحيط بجميع الخواصّ التي تطابق ما وجدنا بالأرصاد مطابقة صحيحة فاتّفقوا كلّهم عليه ومالوا بأجمعهم إليه وصورة هذا الشكل وهيئته على هذا النحو الذي نحن ذاكروه بعد هذا إن شاء الله تعالى وبه التوفيق.

[٣] فليكن فلك البروج دائرة الف راء طاء و فلك معدّل النهار ألف حاء طاء ومركز العالم نقطة العين وليكن وضع الفلك كما كان قبل الهجرة نحوا من خمسين سنة عربية وذلك عند مولد سيّد الأمم وتحفة العرب والعجم محمد صلّى الله عليه وسلّم فإنّ هذه الحركة إذ ذاك لم تكن محسوسة بل كانت نقطتا رأس الحمل والاعتدال واحدة وكان الفلك يومئذ على أعدل وضع وأحسنه والمذهبان أعني مذهبي الهند والممتحن متّفقان في جميع ما يلزم عن ذلك وتعلّم نقطة على معدّل النهار بعدها من نقطة التقاطع التي هي نقطة الألف قدر أربعة أجزاء وثماني دقائق بتقريب ولتكن نقطة الضاد ونجعلها قطباً و <نقطة الألف بعداً ونحطّ> دائرة ألف باء جيم ذال وكذلك نفعل أيضاً عند رأس الميزان فإنّا نجعل قوس طاء دال من معدّل النهار مستوية لقوس ألف ضاد ونجعل الدال قطباً والطاء (٢٤ظ) بعداً وندير دائرة طاء كاف لام ميم فهاتان الدائرتان سمّيتا دائرتي الإقبال والإدبار لأنّ نصف حركتهما حول معدّل النهار وذلك ما دامت شمالية عنه إنّما هو إلى جهة المشرق وحيث مهبّ القبول ونصف حركتهما الثاني حول معدّل النهار أيضاً وذلك ما دامت جنوبية عنه إنّما هو إلى جهة المغرب حيث مهبّ الدبور[200] ولنصل نقطتي الجيم والألف بنقطتي الطاء واللام بخطّين مستقيمين ينقاطعان على مركز العالم الذي هو نقطة العين وليكن قطب معدّل النهار نقطة النون وليكن قوس نون راء حاء من الدائرة العظيمة التي تمرّ بقطبي معدّل النهار وبقطبي فلك البروج فيكون قوس حاء راء هي الميل الكلّي في ذلك الزمان ونجعل نقطة النون مركزاً ونقدّر في قوس نون راء حاء بعداً مبلغه ثلاثة وعشرون جزءاً وثلاث وأربعون دقيقة وذلك أوسط الميول

[200] معنى هذه العبارة غير واضح وربما يعني أن رأس الحمل شرقي عن نقطة الاعتدال.

الكلّية وهو قوس نون فاء وندير دائرة سين فاء قاف[201] (فتكون هذه الدائرة دائرة اختلاف الميل الكلّي ونجعل نقطة فاء مركزاً وندير دائرة صغيرة نصف قطرها عشر دقائق وعشرون ثانية) فتكون هذه الدائرة أيضاً دائرة اختلاف الميل الكلّي ووجدنا أنّ في الزمان المذكور أعني الذي كان فيه رأس الحمل على دائرة معدّل النهار كان القطب إذ ذاك في جهة البعد الأوسط من دائرة اختلاف الميل الكلّي صاعداً إلى (الـ)بعد[202] الأقرب من قطب معدّل النهار فليكن هناك على نقطة السين ونصل السين بالألف والطاء فتكون كلّ واحدة من هاتين القوسين ربع دائرة فنقطة السين إذا هي على محيط دائرة حاء راء نون سين فتكون قوس نون سين هي بعد ما بين القطبين في ذلك الزمان وهو مقدار الميل الكلّي إذ ذاك ونقطة الراء هي المنقلب الصيفي.

[٤] ولتتحرّك دائرة الإقبال والإدبار بنقطتي رأس الحمل والميزان فتتحرّك كلّ واحدة من هاتين النقطتين مائلة عن معدّل النهار أمّا نقطة رأس الحمل ففي قوس ألف هاء ذال شمالية عن معدّل النهار شرقية عن نقطة الاعتدال وأمّا نقطة رأس الميزان ففي قوس طاء زاي كاف جنوبية عن معدّل النهار غربية عن نقطة الاعتدال وأمّا القطب فيتحرّك صاعداً في دائرة اختلاف الميل الكلّي إلى جهة المشرق في قوس سين صاد ولا تزال الحر(ك)ة متّصلة من أجزاء دائرتي الإقبال والإدبار فتكون إذ ذاك نقطتا الألف والطاء[203] اللتان فرضنا رأسي الحمل والميزان (٢٥و) قد زادا أمّا نقطة رأس الحمل فنقطة الهاء وأمّا نقطة

[201] ينقص بين كلمتي "قاف" و "فتكون" سطر ام سطران حذفهما الناسخ.

[202] "بعد" في المخطوط.

[203] "الضاد" في المخطوط.

رأس الميزان فنقطة الزاي ويكون القطب إذ ذاك قد وافى بعده الأقرب من قطب معدّل النهار وهو نقطة الصاد وتنطبق نقطة الثاء من فلك البروج على نقطة التاء من معدّل النهار وتكون نقطة الثاء إذ ذاك نقطة الاعتدال الربيعي وكذلك تنطبق أيضاً نقطة الشين من فلك البروج على نقطة الغين من معدّل النهار فتكون نقطة الغين نقطة الاعتدال الخريفي وينطبق سطح دائرة حاء نون سين على سطح دائرة حاء نون صاد فتكون قوس نون صاد مقدار الميل الكلّي في ذلك الزمان وذلك كج لج وهو أقلّ الميول الكلّية وأقصرها وتكون نقطة لام ألف هي المنقلب الصيفي في ذلك الزمان أيضاً.

[٥] ثمّ تتّصل الحركة أيضاً فتصير أمّا نقطة رأس الحمل فإلى جهة الذال من قوس ألف هاء ذال وأمّا نقطة رأس الميزان فإلى جهة الكاف في قوس طاء زاي كاف وينحدر القطب نحو البعد الأوسط الثاني إلى أن يوافي أمّا رأس الحمل فنهايته الشمالية التي هي نقطة الذال وأمّا نقطة رأس الميزان فنهايتها الجنوبية التي هي نقطة الكاف ويبلغ القطب في ذلك المدّة قريبا من البعد الأوسط الثاني من دائرة اختلاف الميل الكلّي وتنطبق دائرة حاء نون صاد المارّة بالقطبين على دائرة حاء نون قاف المارّة بهما أيضاً وتكون قوس نون قاف هي مقدار الميل الكليّ في ذلك الزمان ونقطة الظاء نقطة الاعتدال الربيعي ونقطة الواو نقطة الاعتدال الخريفي ونقطة الخاء المنقلب الصيفي.

[٦] ثمّ تتّصل الحركة في ربعي ذال جيم و كاف لام من دائرتي الإقبال والإدبار وينحدر القطب نحو البعد الأبعد من قطب معدّل النهار إلى أن تنطبق أمّا نقطة رأس الحمل فعلى نقطة الجيم وأمّا نقطة رأس الميزان فعلى نقطة اللام المشتركتين لدائرتي الإقبال والإدبار ومعدّل النهار فيكون القطب إذ ذاك أقرب ‹من البعد› الأوسط الأول فيعتدل حينئذ وضع

الفلك ويتّفق جميع المذاهب فيه ويرتفع الخلاف الذي بين أهل الهند وأهل الممتحن جملة (٢٥ظ) ولا يكون إذ ذاك للفلك إقبال ولا إدبار.

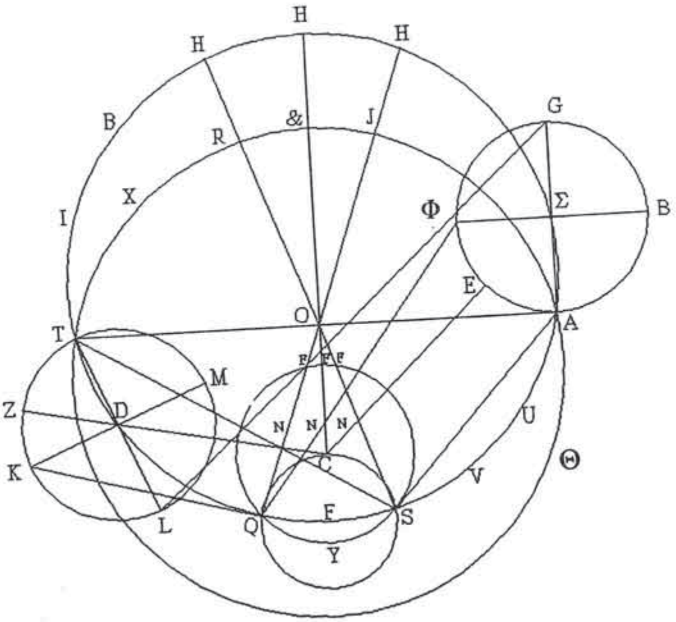
[٧] ثمّ تتّصل الحركة هكذا[204] في نصفي جيم باء ألف ولام ميم طاء من دائرتي الإقبال والإدبار ويصعد القطب إلى جهة بعده الأقرب من قطب معدّل النهار فتتحرّك كلّ واحدة من نقطتي رأس الحمل والميزان في دائرتهما مائلة عن معدّل النهار وأمّا نقطة رأس الحمل فجنوبية عن معدّل النهار غربية عن نقطة الاعتدال وأمّا نقطة رأس الميزان فشمالية عن معدّل النهار وشرقية عن نقطة الاعتدال.

[٨] ويستمرّ الأمر على النحو المتقدّم ذكره بمشيئة الله تعالى إلى أن تعود كلّ واحدة منهما أمّا نقطة رأس الحمل فإلى نقطة الألف وأمّا نقطة رأس الميزان فإلى نقطة الطاء وهما النقطتان المشتركتان لدوائر الإقبال والإدبار ومعدّل النهار اللتان منهما بدآ بالحركة أوّلا وتنطبقان على دائرة معدّل النهار وتكون نقطتا الاعتدال الربيعي ورأس الحمل نقطة واحدة وكذلك نقطتا الاعتدال الخريفي ورأس الميزان نقطة واحدة أيضاً ويعود وضعهما ووضعا نقطتي الانقلابين كما كانا في الزمان الأوّل ويعتدل أيضاً وضع الفلك ولا يوجد له إقبال ولا إدبار وتتّفق المذاهب كلّها ويرتفع الاختلاف جملة وتعود الأمور كلّها كما بدأت أوّل مدّة بتدبير اللطيف الحكيم.

[٩] وفي هذه العودة المذكورة يحدث رأسا الحمل والميزان بحركتيهما على دائرتيهما مخروطي اسطوانة متقابلين متساويين قاعدتاهما سطحا دائرتي الإقبال والإدبار ونقطة رأسيهما مركز العالم وهو نقطة العين وكذلك يحدث القطبان بحركتهما على دائرتي اختلاف الميل الكلّي

[204] "هاكذا" في المخطوط.

وعودتهما فيهما مخروطي اسطوانة أيضاً قاعدتاهما سطحا دائرتي الاختلاف ونقطة رأسيهما مركز العالم فهذا الوضع والشكل هو اللائق من هذه الحركة المفهومة اللازمة عن الاختلاف الموجود بالأرصاد القديمة والحديثة وعلى هذا الأصل نبني العمل في استنباط كمّية مقادير أبعاد رأس الحمل عن نقطة الا<عتدا>ل في جميع الأزمان فلنعلم ذلك <بعدها وبالله التوفيق.



(٢٦و) الباب الثاني.
في كمّية مقادير حركات رأس الحمل والقطب والمركز[205] في دوائرها[206] وأزمان عوداتها[207].

[١] ولمّا نظروا في كمّية هذه المقادير وأزمان العودات اختلفوا في ذلك وكلّ عوّل على ما اقتضته أصوله وأدّاه إليه نظره واجتهاده فأمّا أبو إسحاق إبرهيم بن يحيى رحمه الله فإنّه خرج له بحسب أصوله التي عوّل عليها أمّا نصف قطر دائرة الإقبال فأربعة أجزاء وتسع عشر دقيقة وستّ وعشرون ثانية وثمان ثوالث ⟨...⟩ وهذه المقادير هي أصحّ ما عمل في هذه الحركات عليه و ⟨...⟩ بالجملة إليه ⟨...⟩ إنّه لمّا امتحنّا ما وجدنا له ⟨سردا في الصحف⟩ التي وقعت (٢٦ظ) بأيدينا من مقادير حركتي رأس الحمل والقطب وزماني عودتيهما وجدنا في ذلك من الاختلال بحسب أصوله التي بنا علمها ما أوجب اعادة النظر في تصحيحه لا سيّما حركة القطب فإنّ الذي يخرج فيها من مقادير الميول الكلّية لأزمنة الأرصاد مخالف ما وجد هو وغيره من ذلك بالرصد الصحيح مخالفة ليست باليسيرة ولا مبني ممّا يمكن النتائج فيها ولا التقريب إذ كانت توجب الخلل فيما ينبغى عليها من مقادير أبعاد رأس الحمل عن نقطة الاعتدال.

[205] لا أعيد هنا النصّ الخاصّ بحركة المركز يعني مركز المدير لمركز الفلك الخارج المركز للشمس.

[206] "هما" في المخطوط.

[207] "حركة رأس الحمل والقطب والمركز في دوائرهما (كذا) على استواء في سنة سنة ومن سني التواريخ الثلاثة" في العنوان الذي يظهر في فهرس المقالة.

[٢] فأعدنا نحقّ عمل ذلك والنظر فيه وحققنا النسب التي بها استخرج هو مقادير تلك الازمان والحركات بحسب ما أمكن فكان نسبة الزمان الذي بين بطلميوس وجابر إلى الزمان الذي بين جابر والزرقالة كنسبة ثلاثة أجزاء وخمسين دقيقة وتسع وعشرين ثانية ج ن كط إلى جزء واحد.

[٣] ثمّ لم نزل نحرّك رأس الحمل والقطب على محيطي دائرتيهما تحركا آخر ونطلب اتفاق تلك النسب من مقادير قسيها إلى أن عثرنا بفضل الله تعالى وعونه على ذلك واتّفقت لنا نحقّ في غير المواضع التي اتّفقت له هو فيها وخرج لنا بحسب ذلك أمّا زمان عودة رأس الحمل في دائرته فثلاثة آلاف سنة وثمان مائة سنة وأربع وسبعون سنة رومية كلّها وثلاثة أشهر رومية ونحو من نصف شهر بالتقريب وأمّا عودة القطب في دائرة اختلافه فألفا سنة فارسية واثنان وثلاثون سنة فارسية ونحو من تسعة وعشرين يوما بالتقريب.

[٤] تكون بحسب ذلك حركة رأس الحمل أمّا في سنة رومية فخمسة دقائق وأربعا وثلاثون ثانية[208] وثلاثين ثالثة وخمسا وأربعين رابعا وأربعين خامسة بتقريب ه لد ل مه م وأمّا في سنة فارسية فخمسة دقائق أيضا وأربع وأربعين ثانية وسبعة عشر ثالثة ورابعة واحدة وخُمْسَيْ رابعة ه مد يز اكد وأمّا في سنة عربية فخمس دقائق وأربع وعشرون ثانية وإثني وثلاثين ثالثة وثلاثا وأربعين رابعة ه كد لب مج ⟨و⟩في يوم واحد أربعا وخمسين ثالثة وسبعا وخمسين رابعة و⟨ثلاث خوامس⟩ ٠٠ ند نز ج وتكون ⟨بحسب ذلك حركة⟩ القطب أمّا في سنة رومية فعشر دقائق وثمانية وثلاثين ثانية (٢٧و) وأربع ثوالث وثلاثا

[208] "دقيقة" في المخطوط.

وأربعين رابعة ونحوا من ثلاث خوامس يَ لَحَ دَ مَجَ جَ وأمّا في سنة فارسية فعشر دقائق وسبعا وثلاثين ثانية وثمانيا وثلاثين ثالثة وثلاثين رابعة وخمسا وأربعين خامسة وثمانيا وأربعين سادسة يَ لَزَ لَحَ لَ مَهَ مَحَ وأمّا في سنة عربية فعشر دقائق وتسع عشرة ثانية وثلاث ثوالث وستّ وخمسين رابعة وستّ عشر خامسة وثمانيا وأربعين سادسة بتقريب[209] وهين وفي يوم واحد ثانية واحدة وأربعا وأربعين ثالثة وتسعا وأربعين رابعة ٠ آ مدَ مطَ وبهذه الحركة توافق ما يخرج فيها من مقادير الميول الكلّية جميع الميول المرصودة ميلا ميلا ولا يختلفان البتّة ولا في شيء محسوس أصلا [...] فهذه أزمان الحركات الصحيحة عبّرنا بالأصول الموضوعة لها وبالله التوفيق.

(٢٧و) الباب الثالث.

في أنّه لا يمكن ضبط مقادير أبعاد رأس الحمل عن نقطة الاعتدال[210] الربيعي في جدول يعمّ جميع ما يأتي من الأزمان.

[١] ⟨فو⟩جدنا بحسب ما تقدّم من مقادير حركتي رأس الحمل والقطب أنّ نسبة حركة القطب في دائرة اختلافه إلى حركة رأس الحمل في دائرته كنسبة (٢٧ظ) جزء واحد وأربع وخمسين دقيقة وثلثي دقيقة إلى جزء واحد بتقريب فيلزم عن ذلك أنّ نسبة ما نقصت حركة القطب عن

[209] الحروف الأبجدية ملغاة.

[210] "في جدول عام بجميع الأزمان" في العنوان الذي يظهر في فهرس المقالة.

جزءين كاملين وذلك خمس دقايق وثلث دقيقة إلى ستّين دقيقة كنسبة ما نقص عن عودتين تامّتين إلى عودة واحدة.

[٢] فإذا حملنا العودة الواجدة ستّين ثمّ قسمناها على الخمس الدقائق والثلث دقيقة خرج لرأس الحمل إحدى عشرة عودة وربع عودة ففي الزمان الذي يعود فيه رأس الحمل في دائرته عدد هذه العودات المذكورة إحدى عشر عودة وربع عودة فيه نفسه يعود القطب في دائرته إحدى[211] وعشرين عودة ونصف عودة فإذا بسطنا عددي هذه العودات حصل أمّا رأس الحمل فخمس وأربعين عودة وأمّا القطب فستّ وثمانون عودة وهذان العددان متباينان فهما أقلّ عددين على نسبتهما وأيضاً فإذا إن قسمنا العودة الواحدة على ستّين وضربنا الخارج في خمس وثُلث اجتمع من ذلك اثنان وثلاثون جزءاً وذلك ما تنقص حركة القطب عن عودتين في زمان عودة رأس الحمل فإذا قسمنا على ذلك عودة تامّة خرج لرأس الحمل إحدى عشرة عودة وربع عودة فهي الذي يعود رأس الحمل في دائرته عدد هذه العودات فيه يعود القطب في دائرته إحدى وعشرين عودة ونصف عودة على ما تقدّم وأيضاً فإذا إن بسطنا الجزء الواحد الذي يتحرّك رأس الحمل والجزء الواحد والأربع وخمسين دقيقة وثُلثين دقيقة التي يتحرّكها القطب في ذلك الزمان كان مبسوط الجزء الواحد مائة وثمانين جزءاً ومبسوط الجزء الواحد والأربع والخمسين دقيقة وثلثين دقيقة ثلاثمائة جزء وأربعاً وأربعين دقيقة ففي الزمان الذي يعود فيه رأس الحمل في دائرته مائة وثمانين عودة فيه يعود القطب في دائرته ثلاث مائة عودة وأربعاً وأربعين عودة لاكن هذين العددين ليسا أقلّ عددين على نسبتهما لأنّهما مشتركان فإذا

[211] "اثنتين" في المخطوط وربما هذا خطأ من الناسخ فيجب أن تكون "إحدى".

خزلناهما حصل ⟨أ⟩مّا لرأس الحمل فخمس وأربعين عودة على ما تقدّم وأمّا للقطب[212] فستّ وثمانون عودة وهما على ⟨...⟩ أقلّ جزئين (٢٨و) على نسبت⟨ه⟩ما فيجب إذاً في ذلك أنّه إذا بدأ بالحركة كلّ واحد من رأس الحمل والقطب معاً من نقطتين في دائرتهما في زمان واحد بعينه فإنّهما لا يعودان معاً إلى تينك النقطتين اللتين منهما بدأ بالحركة معاً ولا في مرّة طائلة وذلك بعد أن يكملا في عدة العودات أمّا رأس الحمل فخمسا وأربعين عودة وأمّا القطب فستّا وثمانين عودة وهما أقلّ العودات التي يعودانها معاً في مرّة واحدة فالجدول الذي يعمل لأبعاد رأس الحمل عن نقطة الاعتدال عاماً لجميع الأزمان إنّما يصحّ عمله لمثل عدد هذه العودات المذكورة لرأس الحمل هذا شيء يكاد أن يكون على البشر ممتنعا ولا يوجد أحد به مضطلعاً.

[٣] ولهذه العلّة امتنع أن يدوّن مقادير أبعاد رأس الحمل عن نقطة الاعتدال في جدول يعمّ جميع ما يأتي في الأزمان كما فعل في غيرها من الحركات و مقادير سائر الاختلافات وهذا هو الذي فهمه الجماعة: أبو إسحاق وأبو مروان والفقيه القاضي أبو القاسم صاعد وسائرهم رحمهم الله في كمّية مقادير هاتين الحركتين أعني حركتي رأس الحمل والقطب ولذلك لم يوجد ولا لواحد منهم جدول معمول لها.

[٤] ولمّا جاء المتأخرون بعدهم جهلوا ذلك من هاتين الحركتين فراموا الاضطلاع بما ظنّوا أنّ أولائك لم يقرّروا عليه فوقعوا في الخطاء فيما لم تكن بهم حاجة إليه فقد تبيّن ممّا ذكرنا فساد اعتقاد أبي العباس الكمّاد رحمه الله ومن جزأ جزؤه من أهل زماننا هذا في تشابه حركتي رأس الحمل والقطب في دائرتيهما وامتناع ضبط مقادير الإقبال والإدبار

212 "القطب" في المخطوط.

في جدول عامّ لجميع الأزمان وذلك ما أردنا بيانه.

(٢٨و)الباب الرابع.

في بيان غلط أبي العبّاس الكمّاد رحمه الله في اعتقاده عموم الجدول الذي عمله[213] لحركة الإقبال والإدبار على تشابه حركتي رأس الحمل والقطب لتسعين درجة فقط.

[١] و⟨...⟩ لا يصحّ ⟨عمل الجدول⟩ أيضاً لمائة وثمانين درجة ⟨إلاّ بشرط⟩ زائد على تشابه الحركتين ⟨يعني حركتي⟩ رأس الحمل و القطب في دائرتيهما ⟨في أصل⟩ أبي (٢٨ظ) العبّاس رحمه الله.

[٢] ولتكن قطعة من معدّل النهار عليها ألف باء وقطعة من فلك البروج عليها ألف جيم ونعلّم على محيط معدّل النهار نقطة اللام وندير دائرة لبعد الألف عليها حاء كاف وهي دائرة الإدبار والإقبال وليكن قطب فلك البروج نقطة الدال ودائرة دال ميم دائرة اختلاف الميل الكلّي ولتكن كلّ واحدة من قوسين ألف باء و ألف جيم ربع دائرتها ويخرج من نقطة الدال عموداً على قوس ألف جيم فستمرّ بنقطتي الجيم و الباء معاً وليكن قوس دال جيم فبين أنّه على قوس دال جيم يكون قطب دائرة ألف باء التي هي معدّل النهار فلتكن نقطة النون فنقطة النون هي قطب معدّل النهار.

[٣] فلأنّه زعم أنّ حركة رأس الحمل في دائرة ألف حاء كاف شبيه بحركة القطب في دائرة دال ميم يكون زمان عودتهما واحداً وأنّه إذا كان

[213] "وضعه لذلك." في العنوان الذي يظهر في فهرس المقالة (٢٥ظ).

رأس الحمل على دائرة معدّل النهار كان القطب إذ ذاك في بعده الأوسط من دائرة اختلافه فيلزم أنّه متى تحرّك رأس الحمل في دائرته قوساً ما تحرّك القطب في دائرته قوساً أخرى شبيهة بها.

[٤] فليكن رأس الحمل نقطة الألف المشتركة للفلكي البروج ومعدّل النهار فإذا تحرّكت نقطة الألف في دائرتها قوس ألف حاء وذلك ربع دائرتها تحرّك القطب في دائرته قوس دال طاء ففي الزمان الذي يكون فيه نقطة رأس الحمل في نهايتها الشمالية وهي نقطة الحاء فيه يكون القطب في بعده الأبعد عن نقطة النون وهو نقطة الطاء فيكون رأس الحمل في هذا الزمان قد استوفى جميع ميوله الكلّية عن معدّل النهار ولم يستوف القطب من أبعاده عن نقطة النون الذي هو قطب معدّل النهار الاّ نصفها فقط فلا يصحّ إذاً عموم ذلك الجدول لجميع الأزمان إذ كان القطب قد نقصه من أبعاده عن قطب معدّل النهار الذي هو نقطة النون حركته في قوس هاء دال على دائرة اختلافه وأيضاً فانّه إنّ بني عمل الجدول على هذا ⟨الوجه⟩ لنصف دائرة أعني لمائة وثمانين جزءا ⟨فيكون⟩ رأس الحمل في ⟨طرف هذا الزمان في نقطة الحاء⟩ من معدّل النهار ويكون القطب ⟨إذ ذاك في بعده⟩ (٢٩و) الأوسط الثاني وانطبق على نقطة الميم من دائرة اختلافه ولم يستوف أيضاً في حركته على قوس طاء ميم جميع أبعاده عن قطب معدّل النهار الذي هو نقطة النون ولا كان لتحريكه في هذا الزمان الثاني على قوس طاء ميم فائده ولا زوج سوى تكرير أبعاده الأولى[214] التي كانت له في زمان حركته على قوس دال طاء إذ كانت أبعاده عن نقطة النون في تينك القوسين أعني قوس دال طاء وطاء ميم أبعاد متساوية فلا بدّ إذاً من شرط زائد على ذلك وهو إن يزاد

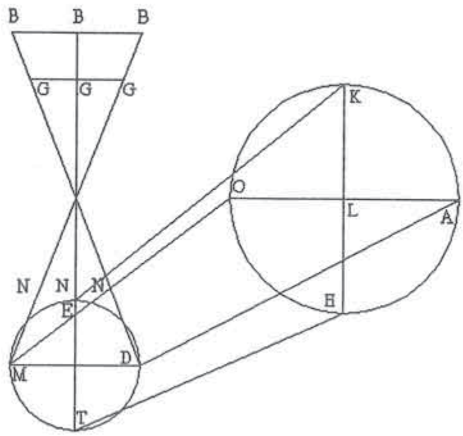
[214] "الأول" في المخطوط.

إلى تلك الأبعاد أبعاده أيضاً على نقطة النون في قوس هاء دال وحينئذ يستوفي القطب في حركته جميع أبعاده الجزئية عن قطب معدّل النهار هو نقطة النون.

[٥] ووجه العمل في ذلك أن تسير نقطة الألف التي هي رأس الحمل إلى وراء قدر زمان عودتها فيكون في طرف ذلك الزمان على نقطة الكاف حيث نهاية ميلها الجنوبي عن معدّل النهار ويكون القطب أيضاً في هذا الزمان قد صعد إلى نقطة الهاء حيث بعده الأقرب إلى قطب معدّل النهار الذي هو نقطة النون.

[٦] ثمّ نجعل مبدأ الجدول من هنالك وتتحرّكان معاً من تينك النقطتين حتى تلحقا أمّا رأس الحمل فبنقطة الحاء حيث نهاية ميوله الشمالية عن معدّل النهار وأمّا القطب فبنقطة الطاء حيث بعده الأبعد عن قطب معدّل النهار فيكون القطب في هذا الزمان قد استوفى جميع أبعاده الجزئية عن قطب معدّل النهار الذي هو نقطة النون ولذلك يكون فلك البروج قد استوفى جميع ميوله الكلّية عن معدّل النهار فهذا الوجه يصحّ عمل الجدول المذكور لمائة وثمانين جزءاً من أجزاء دائرة الإقبال والإدبار ويكون عاما بجميع ما يأتي من الأزمان وحينئذ يجب أن يوضع في أعلاه ستّة بروج وفي أسفله ستّة بروج على ما جرى به العادة في غير ذلك من الاختلافات الجزئية.

[٧] فقد بيّن من ذلك أنّه لا يصحّ ⟨...⟩ أصله الذي هو يشابه الحركتين على جدول عام لجميع ⟨الأزمان ...⟩ جزءاً من أجزاء ⟨دائرة الإقبال⟩ والإ⟨دبار⟩ لمائة وثمانين جزءاً (٢٩ظ) إلاّ أن يكون في طرف ذلك الزمان أمّا رأس الحمل ففي أحد من نهايتي ميله الجنوبي أو الشمالي عن معدّل النهار وأمّا قطب فلك البروج ففي أحد بعديه عن قطب معدّل النهار وأمّا الأبعد أو الأقرب وذلك ما أردنا بيانه.



المقال الثالثة

(٣٥و) الباب الأوّل.
في معرفة ميل فلك البروج الكلّي من قبل الجدول[215].

[١] إذا أردت ذلك فاستخرج حركة القطب الوسطى للوقت الذي تريد ولأيّ تاريخ شئت على ما تقدّم ثمّ ادخل بها في جدول ميول فلك البروج الكلّية بالبروج في عرض الجدول وبالدرج التامّة في طوله وخذ ما بحيال

[215] "ميل البروج الكلّي من قبل الجداويل" في العنوان الذي يظهر في فهرس المقالة.

ذلك من درج الميل ودقائقه وثوانيه واحفظه فإن كان مع درج الحركة الوسطى دقائق فأثبتها ناحية ثمّ اعلم ما يجب لها من التعديل على ما تقدّم. فما كان فاحمله على ما أخذت أوّلا من درج الميل ودقائقه وثوانيه إن كان الميل متزايدا[216] وانقصه منها إن كان متناقصا[217] فما كان الميل الذي أخذت أوّلا بعد الزيادة أو النقصان فهو ميل فلك البروج الكلّي في ذلك الزمان الذي عملت عليه ما عملت فاعلمه وبالله التوفيق.

(٣٥ظ) الباب الثاني.

في معرفة[218] أبعاد رأس الحمل عن نقطة الاعتدال الربيعي من قبل الجدول وهي التي تسمّى[219] حركة الإقبال والإدبار.

[١] إذا أردت ذلك فاستخرج حركة رأس الحمل الوسطى على ما تقدّم للوقت الذي تريد ولأيّ تاريخ شئت على ما تقدّم فما كانت فادخل بها في جدول حركة الإقبال والإدبار بالبروج منها في عرض الجدول وبالدرج التامّة في طوله وخذ ما بحيال ذلك من درج البعد ودقائقه وثوانيه واحفظه فإن كان مع درج الحركة الوسطى دقائق فاعلم ما يجب لها من التعديل على ما تقدّ‹م› فما كان فزده على ما أخذت أوّلا من البعد إن كان

[216] "متزايد" في المخطوط.

[217] "متناقص" في المخطوط.

[218] "نقطة رأس الحمل من نقطة الاعتدال الربيعي من قبل الجدول" في العنوان الذي يظهر في فهرس المقالة.

[219] "تسما" في المخطوط.

البعد متزايدا[220] وانقصه منها إن كان البعد متناقصا[221] فما كان البعد الذي أخذت أوّلا بعد الزيادة عليه أو النقصان منه فهو بعد نقطة رأس الحمل من نقطة ⟨الاعتدال الربيعي من قبل الجدول وهي التي تسمّى حركة الإقبا⟩ل والإدبار في ذلك الزمان الذي ⟨عملت عليه ما عملت⟩.

[٢] (٣٦و) فإن كانت الحركة الوسطى أكثر من ستّة بروج فالبعد الذي خرج لك فيه تقريب يسير وقد تقدّم التنبيه على ذلك في صدر الكتاب وسنذكر بعد هذا إن شاء الله وجه العمل في استخراج بعد نقطة رأس الحمل عن نقطة الاعتدال الربيعي في كلّ زمان على غاية ما يمكن من الصحّة والتحقيق إن شاء الله تعالى وبالله التوفيق.

المقالة السابعة

(٨٠و) الباب الأوّل.

في معرفة أقلّ الميول الكلّية وأبعاد ما بين القطبين من قبل سائر (٨٠ظ) الميول المرصودة[222].

[١] إذا أردت ذلك فاستخرج حركة القطب لزمان الرصد المقصود إليه فما كانت فخذ جيبها واضربه في عشر ثواني وثلث ثانية فما اجتمع فقوّسه تقويس الجيوب فما كانت القوس فخذ جيب تمامها واتخذه إماماً ثمّ خذ جيب تمام الميل المرصود واضربه في ستّين واقسم المجتمع على

[220] "متزايد" في المخطوط.

[221] "متناقص" في المخطوط.

[222] "من قبل الميول المرصود" في العنوان الذي يظهر في فهرس المقالة.

الإمام فما خرج فقوّسه تقويس الجيوب فما كانت القوس فاحفظه ثمّ ارجع إلى حركة القطب فخذ وترها إن كانت أقلّ من مائة وثمانين جزءا أو وترها ينقصها عن ثلاث مائة وستين إن كان أكثر من مائة وثمانين فما كان الوتر فاضربه في عشر ثواني وثلث فما اجتمع فقوّسه تقويس الأوتار فما كانت القوس فخذ جيب تمامها واضربه في ستّين واقسم المجتمع على الإمام فما خرج فقوّسه تقويس الجيوب فما كانت القوس فاسقطها من القوس المحفوظة فما بقي فهو أقلّ الميول الكلّية وأبعاد ما بين القطبين[223] فاعلمه وبالله التوفيق.

[٢] والعلّة في ذلك لتكن دائرة اختلاف الميل الكلّي دائرة ألف باء جيم ولنجرِ على قطبها قوسا من دائرة عظيمة وهي قوس باء الف زاي ولتكن نقطة الزاي منها قطب معدّل النهار فيكون نقطة الألف من دائرة الاختلاف البعد الأقرب[224] إلى قطب معدّل النهار ونقطة الباء منها البعد الأبعد منه ولتكن نقطة الجيم من دائرة اختلاف الميل قطب فلك البروج في زمان رصدها من أرصاد الميل الأعظم ولنجرِ أيضاً عليه وعلى قطب معدّل النهار أعني نقطة الزاي قوسا من دائرة عظيمة وهي قوس زاي جيم فتكون هذه القوس أعني قوس زاي جيم بعد ما بين القطبين ومقدار الميل الأعظم في ذلك الزمان ويكون قوس ألف زاي أقلّ الميول الكلّية وأبعاد ما بين القطبين.

[٣] فأقول إنّها معلومة فلنجرِ على نقطتي الألف والجيم قوساً من دائرة عظيمة وهي قوس ألف جيم ونخرج من نقطة الجيم قوساً من دائرة عظيمة عمودا على قوس ألف باء وهي قوس جيم دال.

[223] "القطب" في المخطوط.

[224] كلمة "الأقرب" ليست في المخطوط.

[٤] فلأنّ قوس ألف جيم من دائرة الاختلاف معلومة تكون قوس باء جيم أيضاً معلومة فجيبها معلوم بالمقدار الذي به نصف قطر دائرة ألف باء جيم ستّون ونصف قطر ⟨دائرة ألف باء جيم⟩ معلوم بالمقدار الذي ⟨به⟩[225] نصف قطر معدّل النهار ⟨ستّون⟩ فقوسه ⟨...زاي جيم⟩[226] (٨١و) بعد ما بين القطبين معلومة بالرصد.

[٥] فمثلّث زاي جيم دال من قسي دوائر عظام وزاوية الدال منه قائمة تكون نسبة جيب تمام قوس جيم زاي إلى جيب تمام قوس زاي دال كنسبة جيب تمام قوس جيم دال إلى جيب ربع الدائرة وكلّ واحد من جيب تمام زاي جيم و جيم دال وجيب ربع الدائرة معلوم فجيب تمام قوس زاي دال معلوم فقوس زاي دال معلومة وأيضاً.

[٦] فلأنّ قوس ألف جيم من دائرة الاختلاف معلومة كما تقدّم يكون وترها معلوما على ما تقدّم بالمقدار الذي به نصف قطر دائرة ألف باء جيم ستّون ونصف قطر دائرة ألف باء جيم معلوم بالمقدار الذي به نصف قطر معدّل النهار ستّون فوتر قوس ألف جيم بذلك المقدار معلوم فالقوس التي عليه من الدائرة العظمى وهي قوس ألف جيم معلومة وقوس جيم دال معلومة.

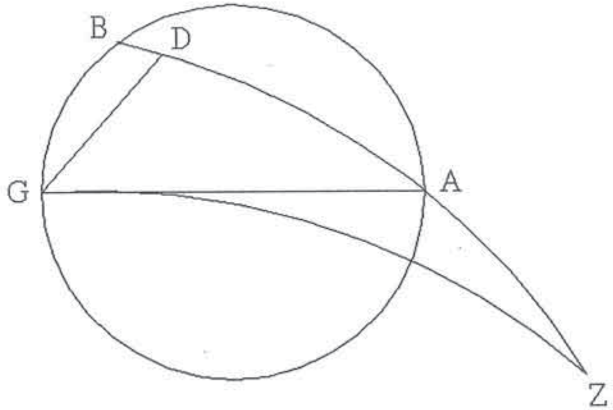
[٧] فمثلّث ألف جيم دال أيضاً من قسي دوائر عظام وزاوية الدال منه قائمة تكون نسبة جيب تمام قوس ألف جيم إلى جيب تمام قوس جيم دال كنسبة ⟨جيب⟩[227] تمام قوس ألف دال إلى جيب ربع الدائرة وجيب تمام قوس ألف جيم معلوم لأنّها معلومة وجيب تمام قوس جيم دال معلوم لأنّها

[225] "به" ينقص في المخطوط.

[226] ينقص هنا سطر مغطى برقعة من قبل مرمّم المخطوط.

[227] ينقص كليمة "جيب" في المخطوط.

معلومة وجيب ربع الدائرة معلوم وذلك ستّون يكون جيب تمام قوس ألف دال معلوما فقوس ألف دال معلومة وقد كانت قوس دال زاي كلّها معلومة فتبقى قوس ألف زاي معلومة وهي أقلّ الميول الكلّية وأبعاد ما بين القطبين وذلك ما أردنا بيانه.



الباب الثاني.

في معرفة مقادير الميول الكلّية وأبعاد ما بين القطبين في كلّ زمان.

(٨١و) [١] إذا أردت ذلك فاستخرج حركة القطب للزمان الذي تريد معرفة الميل الأعظم فيه فإن كانت أقلّ من مائة وثمانين فخذ وترها

وجيب ما <ينقصها عن>[228] مائة وثمانين و<إن كانت>[229] أكثر من <مائة وثمانين>[230] فاسقطها من ثلاثة مائة (١٨ظ) وستّين فما بقي فخذ وتره وجيب ما ينقصه عن مائة وثمانين فما كان من الوتر والجيب معاً فاضرب كلّ واحد منهما[231] في عشر ثواني وثلث ثانية فما اجتمع له فقوّسه تقويس جنسه فما كانت قوس الجيب فخذ جيب تمامها واتّخذه إماما وما كانت قوس الوتر فخذ جيب تمامها واضربه في ستّين واقسم المجتمع على الإمام فما خرج فقوّسه تقويس الجيوب ⟨التمام⟩[232] فما كانت القوس فاحمل عليها أقلّ الميول الكلّية وقد تقدّم العلم بها فما اجتمع فخذ جيب تمامه واضربه فيما اتّخذ به إماما واقسم المجتمع على ستّين فما خرج فقوّسه تقويس جيوب التمام فما كانت القوس فهي الميل الكلّي وبعد ما بين القطبين في ذلك الزمان فاعلمه وبالله التوفيق.

[٢] والعلّة في ذلك لنعدّ الشكل المتقدّم على هيئته وصورته فيكون قوس زاي جيم على ما تقدّم مقدار الميل الكلّي وبعد ما بين القطبين في الزمان المفروض فأقول إنّها معلومة فلأنّ قوس ألف جيم من دائرة ألف باء جيم وهي دائرة الاختلاف معلومة ممّا دوّن في الجداول تكون أيضاً قوس باء جيم معلومة ولذلك يكون كلّ واحد من وتر قوس ألف جيم

[228] من "الزيج القويم" لابن الرقّام (٢٨و). انظر عبد الرحمان، ١٩٩٦، ١١١.

[229] من "الزيج القويم" لابن الرقّام (٢٨و). انظر عبد الرحمان، ١٩٩٦، ١١١.

[230] من "الزيج القويم" لابن الرقّام (٢٨و). انظر عبد الرحمان، ١٩٩٦، ١١١.

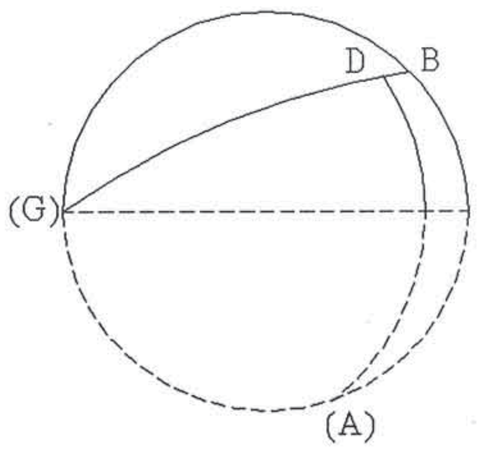
[231] "من منهما" في المخطوط وفي "الزيج القويم" لابن الرقّام (٢٨و). "كل واحد منهما". انظر عبد الرحمان، ١٩٩٦، ١١١.

[232] ينقص كليمة "التمام" في المخطوط.

وجيب باء جيم معلوماً بالمقدار الذي به نصف قطر دائرة باء جيم ألف ستّون فكلّ واحد منها إذا على ما تبيّن معلوم بالمقدار الذي به نصف قطر معدّل النهار ستّون فالقوسان اللّتان عليهما من الدائرتين العظيمتين أعني قوس ألف جيم وجيم دال معلومتان.

[٣] فمثلّث ألف جيم دال من قسي دوائر عظام وزاوية الدال منه قائمة وضلعا ألف جيم وجيم دال منه معلومان يكون ضلع ألف دال الباقي منه معلوما وقوس ألف زاي على ما تبيّن معلومة تكون قوس زاي دال كلّها معلومة.

[٤] فمثلّث دال زاي جيم أيضاً من قسي دوائر عظام وزاوية الدال منه قائمة وضلعا زاي دال ودال جيم منه معلومان يكون ضلع زاي جيم الباقي منه معلوما وهو بعد ما بين القطبين ومقدار الميل الكلّي وذلك ما أردنا بيانه.



(٨٢و) الباب الثالث.

في معرفة أبعاد نقطة رأس الحمل عن نقطة الاعتدال الربيعي في كلّ زمان وهو الإقبال والإدبار المحسوس[233].

[١] ولما كانت هذه الأبعاد الموجودة لرأس الحمل عن نقطة الاعتدال في فلك البروج تختلف أيضاً باختلاف حركة القطب على ما قد بينّاه وجب أن نبيّنه على وجه العمل في استخراج مقاديرها في كلّ زمان من الأزمان الآتية بعد بحول الله تعالى فإذا أردت ذلك فاستخرج حركة رأس الحمل الوسطى لذلك الزمان الذي تريد فما كانت فهي الحصّة فادخل بها في جدول جيوب ميول رأس الحمل وخذ ما بحيالها من جيب الميل فإن كانت الحصّة من درجة إلى مائة وثمانين فالميل شمالي وإن كانت بخلاف ذلك فالميل جنوبي فما كان جيب الميل فاضربه في ستّين فما اجتمع فاقسمه على جيب الميل الكلّي في ذلك الزمان فما خرج فقوّسه تقويس[234] الجيوب فما كانت القوس فهو بعد رأس الحمل عن نقطة الاعتدال الربيعي في ذلك الزمان وهو الإقبال الأول والإدبار فإن كانت حصّة رأس الحمل من درجة إلى تسعين أو من مائتين وسبعين إلى ثلاث مائة وستين فهو مقبل نحو الشمال وإن كانت بخلاف ذلك فهو مقبل نحو الجنوب وأيضاً فإنّه إن كان الميل شمالياً فإنّ الإقبال والإدبار شرقي عن نقطة الاعتدال الربيعي ونقطة رأس الحمل متقدّمة لها نحو المشرق وإن كان الميل جنوبياً فإنّ الإقبال والإدبار غربي عن نقطة الاعتدال الربيعي

[233] "وهو الإقبال الأول المحسوس" في العنوان الذي يظهر في فهرس المقالة.

[234] "تقويم" في المخطوط.

ونقطة رأس الحمل متأخـّرة عنها نحو المغرب.

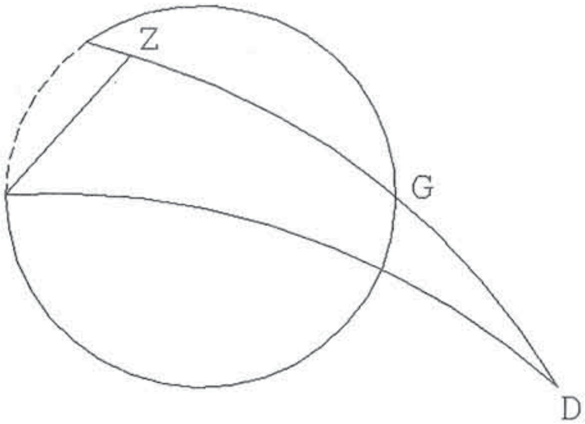
[٢] والعلـّة في ذلك لتكن دائرة الإقبال والإدبار دائرة الف باء جيم حول قطر الف جيم ولنجرّ على قطبها وعلى نقطتين الالف والجيم قطعة من معدّ⟨ل⟩ النهار وهو قوس الف جيم دال ولتكن نقطة رأس الحمل على نقطة الباء من دائرة الف باء جيم ونخرج منها قوساً من دائرة عظيمة عموداً على قوس الف جيم وهو قوس باء زاي ونجرّ على نقطتي الباء والدال قطعة من فلك البروج وهي قوس دال باء فتكون نقطـ⟨ة⟩ الدال ⟨مشتركة لمـ⟩عدّل النهار وفلك البروج ⟨وهو الا⟩عتدال الربيعي ⟨في⟩ ذلك الزمان فـ⟨تكون قوس باء دال وذلك⟩ (٨٢ظ) بعد نقطة رأس الحمل عن نقطة الاعتدال الربيعي وذلك مقدار الإقبال الأول المحسوس في ذلك الزمان.

[٣] فأقول إنـّها معلومة[235] فلأنّ مثلـّث دال زاي باء من قسي دوائر عظام تكون نسبة جيب قوس ⟨باء زاي إلى جيب قوس⟩[236] باء دال كنسبة جيب قوس زاوية الدال إلى جيب قوس زاوية الزاي.

[٤] وجيب قوس باء زاي معلوم لأنها ميل رأس الحمل عن معـ⟨دّ⟩ل النهار وجيب قوس زاوية الدال معلوم لأنها الميل الأعظم في ذلك الزمان وجيب قوس زاوية الزاي معلوم لأنها ربع الدائرة فيكون جيب قوس دال باء معلوماً فقوس دال باء معلومة وهي مقدار بعد رأس الحمل عن نقطة الاعتدال الربيعي في ذلك الزمان وهو الإقبال الأوّل المحسوس وذلك ما أردنا بيانه فاعلمه وبالله التوفيق.

[235] يعني قوس باء دال.

[236] ينقص في المخطوط.



الباب الرابع.
في معرفة مطالع أجزاء دائرة الإقبال[237] في دائرة نصف النهار وهو الإقبال الثاني المعقول.

[١] إذا أردت ذلك فاستخرج حركة رأس الحمل الوسطى لذلك الزمان الذي تريد فما كانت فهي الحصّة فإن كانت أقلّ من مائة وثمانين فخذ وترها وجيب ما ينقصها عن مائة وثمانين وإن كانت أكثر من مائة

[237] "أجزاء الإقبال" في العنوان الذي يظهر في فهرس المقالة.

وثمانين فانقصها من ثلاث مائة وستين فما بقي فخذ وتره وجيب ما ينقصه على مائة وثمانين فما كان من الوتر والجيب معاً فاضرب كلّ واحد منها في أربع دقائق وتسع عشر ثانية وستّ وعشرين ثالثة فما اجتمع فقوّسه تقويس جنسه فما كانت قوس الجيب فخذ جيب ⟨تمامها واتّخذه إماما وما كان قوس الوتر فخذ جيب تمامها واضربه في ستّين واقسم المجتمع على الإمام فما خرج فقوّسه تقويس الجيوب[التمام][238] فما كانت⟩[239] (٨٣و) القوس فهي الإقبال الثاني وتلك مطالع ما تحركه رأس الحمل عن معدّل النهار في دائرة الإقبال فاعلمه وبالله التوفيق.

[٢] والعلّة بيّنة ممّا تقدّم إلاّ أنّ ها هنا ما يوجب اعاده القول في ذلك فلتكن دائرة الإقبال والإدبار دائرة الف باء جيم حول مركز الهاء وقطر الف جيم ولنجر على قطبها وعلى نقطتي الالف والجيم قوساً من دائرة معدّل النهار وهي قوس الف جيم ولتكن نقطة الباء من دائرة الإقبال موضع رأس الحمل منها في الزمان المفروض ونخرج منها عموداً على قوس معدّل النهار وهي قوس باء زاي فتكون قوس باء زاي ميل رأس الحمل في ذلك الزمان عن معدّل النهار ونجر على نقطتي الجيم والباء قوساً من دائرة عظيمة وهي قوس باء جيم ونصل الجيم بالباء بخطّ مستقيم وهو خطّ جيم باء فلأنّ قوسي جيم باء وباء الف من دائرة الإقبال معلومتان يكون كلّ واحد من وتر جيم باء وجيب قوس الف باء معلوماً بالمقدار الذي به نصف قطر دائرة الإقبال ستّون ونصف قطر دائرة الإقبال معلوم بالمقدار الذي به نصف قطر معدّل النهار

[238] "التمام" تصحيح عبد الرحمن على نص ابن الرقّام .

[239] السطران الأخيران مغطى برقعة من قبل مرمّم المخطوط والنصّ هنا من "الزيج القويم" لأبن الرقّام (٢٨ظ). انظر "حساب أطوال الكواكب" و١١٢-٣١١.

ستّون فكلّ واحد اذاً من وتر جيم باء وجيب قوس الف باء معلوم بالمقدار الذي به نصف قطر معدّل النهار ستّون فالقوسان اللتان على وتر جيم باء وجيب قوس الف باء من الدائرتين العظيمتين وهما قوسا جيم باء وباء زاي معلومان.

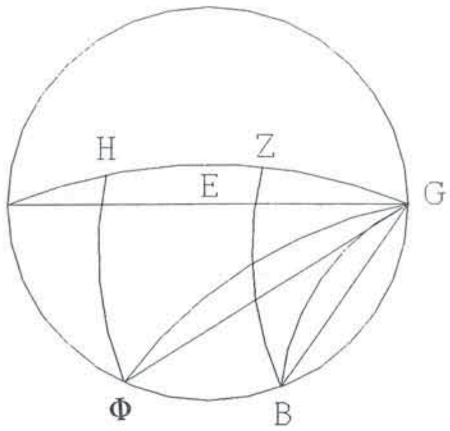
[٣] فيكون مثلّث جيم باء زاي من قسي دوائر عظام وزاوية الزاي منه قائمة فنسبة جيب تمام قوس باء جيم إلى جيب تمام قوس جيم زاي كنسبة جيب تمام قوس باء زاي إلى جيب ربع الدائرة وكلّ واحد من جيبي تمام قوسي جيم باء وباء زاي وجيب ربع الدائرة معلوم فجيب تمام قوس جيم زاي معلوم فقوس جيم زاي من معدّل النهار معلومة وهي إقبال رأس الحمل الثاني المعقول وذلك ما يطلع في[240] دائرة نصف النهار من أجزاء معدّل النهار مع قوس جيم باء من دائرة الإقبال.

[٤] وأيضاً فليكن رأس الحمل على نقطة الذال من دائرة الأقبال في زمان ما آخر ونخرج من نقطة ⟨الذال عموداً على قوس معدّل⟩ النهار وهو قوس ⟨ذال حاء فتك⟩ون هذه القوس بين ⟨نق⟩طة رأس الحمل عن معدّ⟨ل النهار ...⟩ما ونجر على نقطتي الذال والجيم قوساً من دائرة عظيمة وهي قوس ذال جيم ونصل الجيم بالذال بخطّ مستقيم وهو خطّ جيم ذال.

[٥] فيكون مثلّث ذال حاء جيم من قسي دوائر عظام وزاوية الحاء منه قائمة تكون بما تقدّم من البرهان قوس جيم حاء من معدّل النهار معلومة وقد كانت قوس جيم زاي منه معلومة فتبقى قوس حاء زاي معلومة وهي مقدار الإقبال الثاني فيما بين الزمانين وذلك ما يجوز في دائرة نصف النهار من معدّل النهار مع قوس ذال باء من دائرة الإقبال التي هي ما

[240] "كذا في المخطوط وفي عنوان الباب في هذا "الزيج" وفي "الزيج القويم" لابن الرقّام ففي نص "الزيج القويم" (٢٨ظ) "وتلك مطالع ما تحرّكه رأس الحمل عن معدّل النهار في دائرة الإقبال".

يتحركه رأس الحمل منها في المدة التي بين الزمانين.



V. References

ABDULRAHMAN [1996a]

ABDULRAHMAN, M., "Wujūd Jadāwil fī Zīj Ibn al-Hā'im", (In Arabic, with a summary in English). *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*. Ed. by J. Casulleras and J. Samsó (Barcelona, 1996) vol I, 365-381.

ABDULRAHMAN [1996b]

ABDULRAHMAN, M., "Ḥisāb Aṭwāl al-Kawākib fī *'l-Zīj al-Shāmil fī Tahdhīb al-Kāmil* li-Ibn al-Raqqām". Unpublished Ph.D. dissertation (in Arabic). University of Barcelona (1996).

AL-BATTĀNĪ

AL-BATTĀNĪ, Muḥammad ibn Jābir Ibn Sinān *Kitāb al-Zīj al-Ṣābi'*,

See Nallino [1899].
AL-BĪRŪNĪ
AL-BĪRŪNĪ, Abū 'l-Rayhān, "Kitāb al-Athār al-Bāqiya", See Sachau [1923].
AL-HASHIMĪ
AL-HASHIMĪ, ᶜAli ibn Sulayman, in *Kitāb fī ᶜIlal al-Zījāt*.In *The Book of the Reasons Behind Astronomical Tables*. Facsimile edition of MS. Bodleian Arch. Seld. A11, translation by F.I. Haddad and E.S. Kennedy and commentary by D. Pingree and E.S. Kennedy (Delmar-New York, 1981).
ALKUWAIFI & RIUS
ALKUWAIFI A. & RIUS, M., "Descripción del MS. 80 de *al-Zāwiya al-Ḥamzawīya*". *Al-Qanṭara*, XIX (1998), 445-463.
CALVO [1997]
CALVO, E., "Ibn al-Kammad's Astronomical Work in Ibn al-Hā'im's *al-Zīj al-Kāmil fī-l-Taᶜālīm*. I. Solar Year and Trepidation", a paper delivered at the XXth International Congress of History of Science (Liège, July 1997).
CALVO [1998]
CALVO, E., "Astronomical Theories Related to the Sun in Ibn al-Hā'im's *al-Zīj al-Kāmil fī-l-Taᶜālīm*". *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 12 (Frankfurt, 1998), 51-111
CHABÁS & GOLDSTEIN [1994]
CHABÁS, J. & GOLSTEIN, B.R., "Andalusian Astronomy: *al-Zīj al-Muqtabis* of Ibn al-Kammād". *Archives for the History of Exact Sciences*, 48 (1994), 1-41.
CHABÁS & GOLDSTEIN [1996]
CHABÁS, J. & GOLSTEIN, B.R., "Ibn al-Kammād's Star List". *Centaurus*, 38 (1996), 317-334.
COMES [1990]
COMES, M., *Ecuatorios Andalusíes: Ibn al-Samḥ, al-Zarqālluh y Abū 'l-Ṣalt*. Barcelona, 1990.
COMES [1991]
COMES, M., "Deux Échos Andalous à Ibn al-Bannā' de Marrākush". *Actes du VII Colloque Universitaire Tuniso-Espagnol sur le Patrimoine*

Andalous dans la Culture Arabe et Espagnole. Tunis 3-10 Fevrier 1989. (Cahiers du CERES, série Histoire n° 4. Tunis, 1991), 81-94.

COMES [1992]
COMES, M. "A propos de l'influence d'al-Zarqālluh en Afrique du Nord: l'apogée solaire et l'obliquité de l'ecliptique dans le *zīdj* d'Ibn Isḥāq". *Actas del "II Coloquio Hispano-Marroquí de Ciencias Históricas: Historia, Ciencia y Sociedad"*, (Madrid, 1992), 147-159.

COMES [1996]
COMES, M., "Accession and Recession Theory in al-Andalus and the North of Africa" *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*. Ed. by J. Casulleras and J. Samsó (Barcelona, 1996), 349-364.

COMES [1997]
COMES, M., "Some New Maghribī Sources dealing with Trepidation", a paper delivered at the XXth International Congress of History of Science (Liège, July 1997). Forthcoming.

COMES [1998]
COMES, M., "The possible scientific exchange between the courts of Hulaghu and Alfonso X" delivered at the 2e Colloque International "La Science dans le Monde Iranien" (Tehran, June 1998). Forthcoming.

CORRIENTE [1977]
CORRIENTE, F., *A Grammatical Sketch of the Spanish Arabic Dialect Bundle*. Madrid, 1977.

CORRIENTE [1992]
CORRIENTE, F., *Árabe Andalusí y Lenguas Romances*. Madrid, 1992.

DOBRZYCKI [1965]
DOBRZYCKI, J., "Teoria precesji w astronomii sredniowiecznei". *Studia i Materialy Dziejow Nauki Polskiej*, Seria Z, Z 11 (1965), 3-47.

FORCADA [1994]
FORCADA, M., "Rīḥ". *Encyclopédie de l'Islam*, 2nd ed., VIII, 544-5.

GIRKE [1988]
GIRKE, D., *Drei Beiträge zu den Frühesten Islamischen Sternkatalogen*. Preprint Series, 8. Institut für Geschichte der Naturwissenschaften (Frankfurt, 1988).

GOLDSTEIN [1964]
GOLDSTEIN, B.R., "On the Theory of Trepidation According to

Thābit b. Qurra and al-Zarqālluh and its Implications for Homocentric Planetary Theory". *Centaurus*, 10 (1964), 232-247

HARTNER [1984]

HARTNER, W., "Trepidation and Planetary Theories. Common Features in Late Islamic and Early Renaissance Astronomy". *Oriens.Occidens*, II (Hildesheim, 1984).

KENNEDY [1956]

KENNEDY, E.S. [1956] "A Survey of Islamic Astronomical Tables". *Transactions of the American Philosophical Society* Philadelphia, 1956.

KENNEDY [1991-1992]

KENNEDY, E.S., "Transcription of Arabic Letters in Geometric Figures". *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 7 (1991-2), pp. 21-22.

KENNEDY [1997]

KENNEDY, E.S., "The Astronomical Tables of Ibn al-Raqqām a Scientist of Granada" *Zeitschrift für Geschichte der Arabisch-Islamischen Wisenschaften*, 11 (1997), 35-72.

KING [1981]

KING D.A., *Fihris al-makhṭūṭāt al-ʿilmiyya al-maḥfūẓa bi-Dār al-Kutub al-Miṣriyya* I, Cairo, 1981.

KING [1986]

KING, D.A., *A Survey of the Scientific Manuscripts in the Egyptian National Library*, Winnona Lake, Indiana, 1986.

KING [1989]

KING, D.A., "Maṭlaʿ". *Encyclopédie de l'Islam*, 2nd ed., VI, 830-1.

KUNITZSCH [1986]

KUNITZSCH, P., *Der Sternkatalog des Almagest. Die arabisch-mittelalterliche Tradition. I Die arabischen Übersetzungen*. Wiesbaden, 1986.

LORCH [1973]

LORCH, R.P., "Jābir Ibn Aflaḥ". *D.S.B.* VII (New York, 1973), 37-39.

LORCH [1975]

LORCH, R.P., "The Astronomy of Jābir ibn Aflaḥ". *Centaurus*, 19 (Copenhagen, 1975), 85-107. Reprinted in *Arabic Mathematical*

Sciences. Instrumments, Texts, Transmission. Variorum (Aldershot, 1995), VI.

LORCH [1995]

LORCH, R.P., "The Manuscripts of Jābir's Treatise" and "Jābir ibn Aflaḥ and the Establishment of Trigonometry in the West" Appendices 1 and 2 to item VI [Lorch:1975]. First publications. *Arabic Mathematical Sciences. Instrumments, Texts, Transmission.* Variorum (Aldershot, 1995).

MANCHA [1998]

MANCHA, J.L., "On Ibn al-Kammād's Table for Trepidation". *Archives for the History of Exact Sciences*, 52 (1998), 1-11.

MANNŪNĪ [1963]

MANNŪNĪ, M., "Maktabat al-Zāwiya al-Ḥamzīya". *Tiṭwān* VIII (1963), 97-177.

MERCIER [1976-1977]

MERCIER, R., "Studies in the Medieval Conception of Precession". *Archives Internationales d'Histoire des Sciences*, 26 (1976), 197-220 and 27 (1977), 33-71 OJO.

MERCIER [1987]

MERCIER, R., "Astronomical Tables in the Twelfth Century". *Adelard of Bath. An English Scientist and Arabist of the Early Twelfth Century.* Ed by Ch. Burnet (London, 1987), 87-118.

MERCIER [1996]

MERCIER, R. [1996] "Accession and Recession: Reconstruction of the Parameters" *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet.* Ed. by J. Casulleras and J. Samsó (Barcelona, 1996), 299-347.

MESTRES [1996]

MESTRES, A., "Maghribī Astronomy in the 13th century: a Description of Manuscript Hyderabad Andra Pradesh State Library 298". *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet.* Ed. by J. Casulleras and J. Samsó (Barcelona, 1996), 383-443.

MESTRES [1999]

MESTRES, A. "An Andalusi *zīj* in 13th Century Tunis". Unpublished Ph.D. dissertation. 2 vols (Barcelona, 1999).

MIELGO [1996]

MIELGO, H., "A Method of Analysis for Mean Motion Astronomical Tables", *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*. Ed. by J. Casulleras and J. Samsó (Barcelona, 1996), pp. 159-179.

MILLAS VALLICROSA [1943-1950]

MILLÁS VALLICROSA, J.M., *Estudios sobre Azarquiel*. Madrid-Granada, 1943-1950.

MOESGAARD [1975]

MOESGAARD, K.P., "Tychonian Observations, Perfect Numbers and the Date of Creation. Longomontanus Solar and Precessional Theories", *Journal for the History of Astronomy* 6 (1975), 84-99.

MORELON [1994]

MORELÓN, R., "T̲ābit b. Qurra and Arabic Astronomy in the 9th Century". *Arabic Sciences and Philosophy*, 4 (1994), 111-139.

NALLINO [1899-1903-1907]

NALLINO, C.A., *Al-Battānī Kitāb al-Zīj al-Ṣābi' (Al-Battānī sive Albatenii, Opus astronomicum)*, edition, translation and commentary. 3 vols. Milan, 1899-1907 (vol. 1 [1903]; vol. 2 [1907]; vol 3 [1899]).

NEUGEBAUER [1962]

NEUGEBAUER, O., "Thābit ben Qurra, *On the Solar Year* and *On the Motion of the Eighth Sphere*". *Proceedings of the American Philosophical Society*, 106 (1962), 264-299.

NORTH [1967a]

NORTH, J.D., "Medieval Star Catalogues and the Movement of the Eighth Sphere". *Archives Internationales d'Histoire des Sciences*, 17 (1967), 73-83.

NORTH [1976b]

NORTH, J.D., "Thebit's Theory of Trepidation and the Adjustment of John Maudith's Star Catalogue". *Richard of Wallingford. An edition of his writings with introductions, English translation and commentary*, 3 vols. Oxford, 1976, vol. III, apendix 25, 155-8.

PINGREE [1972]

PINGREE, D., "Precession and Trepidation in Indian Astronomy before A.D. 1200". *Journal for the History of Astronomy*, 3 (1972), 27-35.

PTOLEMY
PTOLEMY, C. *Almagest*. See Toomer [1984].
PUIG [2000]
PUIG, R., "The Theory of the Moon in the *al-Zīj al-Kāmil fi-l-Taᶜālīm* of Ibn al-Hā'im (*ca*. 1205)". *Suhayl*, 1 (2000), 71-99.
RAGEP [1993]
RAGEP, J., "*Naṣīr al-Dīn al-Ṭūsī*'s Memoir on Astronomy (*al-Tadhkira fī ᶜilm al-Hay'a*)" 2 vols. *Sources in the History of Mathematics and Physical Sciences*, 12. New York, 1993.
RAGEP [1996]
RAGEP, J., "Al-Battānī, Cosmology and the History of Trepidation in Islam" *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*. Ed. by J. Casulleras and J. Samsó (Barcelona, 1996), 267-298.
RASHED [1996]
RASHED, R., *Les mathématiques infinitésimales du IXᵉ au XIᵉ siècle. Fondateurs et commentateurs. Vol 1. Banū Mūsā, Ibn Qurra, Ibn Sinān, al-Khāzin, al-Qūhī, Ibn al-Samḥ, Ibn Hūd*. London, 1996, chapter VI, 885-973.
SACHAU [1923]
SACHAU, E., *Chronologie Orientalischer Völker von Alberuni (al-Āthār al-bāqiya ᶜan al-qurūn al-khāliya)*. London 1878, reprinted in Leipzig, 1923.
ṢĀᶜID AL-ANDALUSĪ [1]
ṢĀᶜID AL-ANDALUSĪ, *Ṭabaqāt al-Umam*. Ed. Cheikho. *al-Mashriq*. Beirut, 1912.
ṢĀᶜID AL-ANDALUSĪ [2]
ṢĀᶜID AL-ANDALUSĪ, *Ṭabaqāt al-Umam*. Tranlated as "Livre des Catégories des Nations" by R. Blachère. Paris, 1935.
ṢĀᶜID AL-ANDALUSĪ [3]
ṢĀᶜID AL-ANDALUSĪ, *Ṭabaqāt al-Umam*. Ed. Ḥayāt Bū ᶜAlwān. Beirut, 1985.
ṢĀᶜID AL-ANDALUSĪ [4]
ṢĀᶜID AL-ANDALUSĪ, *Al-Taᶜrīf bi-Ṭabaqāt al-Umam*. Ed. Gholāmreḍā Jamthīdnežād-e Avval. Tehran, 1997.
ṢĀᶜID AL-ANDALUSĪ [5]

ṢĀᶜID AL-ANDALUSĪ, *El Libro de las categorías de las naciones (Kitāb Ṭabaqāt al-Umam*. Tr. f. Maíllo Salgado, Madrid, 1999.

SALIBA [1994]

SALIBA, G., "Theory and Observation in Islamic Astronomy: The Work of Ibn al-Shāṭir of Damascus".*Journal for the History of Astronomy*, 18 (1987), 35-43. Reprinted in *A History of Arabic Astronomy. Planetary Theories during the Golden Age of Islam*. New York, London, 1994, 233-241.

SALIBA [1996]

SALIBA, G., "Arabic Planetary Theories after the Eleventh Century AD". *Encyclopedia of the History of Arabic Science*. (New York, London, 1996), vol. 1, 58-127.

SALIBA [1999]

SALIBA, G., "Critiques of Ptolemaic Astronomy in Islamic Spain". *Al-Qanṭara* XX, fasc. 1 (1999), 3-25.

SAMSÓ [1980]

SAMSÓ, J., "Notas sobre la trigonometría esférica de Ibn Muᶜāḏ". *Awrāq*, 3 (Madrid, 1980), 60-68. Reprinted in *Islamic Astronomy and Medieval Spain*. Variorum (Aldershot, 1994), VII.

SAMSÓ [1987a]

SAMSÓ, J., "On the Solar Model and the Precession of the Equinoxes in the Alfonsine *Zīj* and its Arabic Sources". *History of Oriental Astronomy*. Proceedings of an International Astronomical Union Colloquium no. 91, held in New Delhi, India (November, 1985), ed G. Swarup, A.K. Bag and K.S. Shukla (Cambridge 1987), 175-183. Reprinted in *Islamic Astronomy and Medieval Spain*. Variorum (Aldershot, 1994), XIX.

SAMSÓ [1987b]

SAMSÓ, J., "Sobre el modelo de Azarquiel para determinar la oblicuidad de la eclíptica". *Homenaje al Prof. Darío Cabanelas O.F.M. con motivo de su LXX aniversario*, II (Granada, 1987), 367-377. Reprinted in *Islamic Astronomy and Medieval Spain*. Variorum (Aldershot, 1994), IX.

SAMSÓ [1992]

SAMSÓ, J. [1992] *Las Ciencias de los Antiguos en al-Andalus*. Madrid,

1992, 219-245.
SAMSÓ [1994a]
SAMSÓ, J., "Trepidation in al-Andalus in the 11th century" in *Islamic Astronomy and Medieval Spain*. Variorum (Aldershot, 1994), VIII.
SAMSÓ [1994b]
SAMSÓ, J. "Ibn al-Bannā', Ibn Isḥāq and al-Zarqālluh's Solar Theory" in *Islamic Astronomy and Medieval Spain*. Variorum (Aldershot, 1994), X.
SAMSÓ [1997]
SAMSÓ, J., "Andalusian Astronomy in 14th century Fez: *al-Zīj al-Muwāfiq* of Ibn ᶜAzzūz al-Qusanṭīnī". *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, band 11 (1977), 108 -109.
SAMSÓ [1998]
SAMSÓ, J., "An outline of the history of Maghribī *zījes* from the end of the thirteenth century". *Journal for the History of Astronomy*, 29 (1998), 93-102.
SAMSÓ & MILLÁS [1998]
SAMSÓ, J. & MILLÁS, E., "The computation of Planetary Longitudes in the *Zīj* of Ibn al-Bannā'". *Arabic Sciences and Philosophy*, 8 (1998), 259-286.
SAMSÓ & BERRANI [1999]
SAMSÓ, J. & BERRANI, H., "World Astrology in eleventh century al-Andalus: The epistle on *Tasyīr* and the Projection of Rays by al-Istijjī". *Journal of Islamic Studies* 10:3 (1999), 293-312.
SAᶜIDĀN [1983]
SAᶜIDĀN, A.S., *Rasā'il Ibn Sinān* (by Sinān b. Thābit) ed. by A.S. Saᶜīdān. Kuwait, 1983.
SINĀN
SINĀN, Ibn Thābit, *Rasā'il*. See Saᶜidān [1983]
SOUISSI [1968]
SOUISSI, M., *La Langue des Mathématiques en Arabe*. Tunis, 1968.
SUTER [1892-1922]
SUTER, H., "Die Mathematiker und Astronomen der Araber und ihre Werke". Reprinted by F. SEZGIN in Frankfurt (1986), *Beiträge zur Geschichte der Mathematik und Astronomie im Islam* (2 vols.), band 1, 1-285.

SWERDLOW & NEUGEBAUER [1984]
SWERDLOW, N.M. & NEUGEBAUER, O., "Mathematical Astronomy in Copernicus's 'De Revolutionibus'" (2 vols.), *Studies in the History of Mathematics and Physical Sciences* 10 (New York, 1984).
TIHON [1978]
TIHON, A., *Le "Petit Commentaire" de Théon d'Alexandrie aux Tables Faciles de Ptolémée.* Vatican, 1978, 236-7 and 319.
TOOMER [1968]
TOOMER, G.J., "A Survey of the Toledan Tables. *Osiris*, 15 (1968), 15-174.
TOOMER [1984]
TOOMER, G.J., *Ptolemy's Almagest*. Translation and notes. New York, 1984.
TOURNEAU [1960]
TOURNEAU, R. le, "Barghawāṭa". *Encyclopédie de l'Islam*, 2nd ed., I, 1075-1076.
VERNET [1952]
VERNET, J., *Contribución al Estudio de la Labor Astronómica de Ibn al-Bannā'*. Tetuan, 1952.
VILLUENDAS [1979]
VILLUENDAS, M.V., *La trigonometría europea en el siglo XI. Estudio de la obra de Ibn Muᶜāḏ El 'Kitāb maŷhūlāt qisī al-kura'*. Barcelona, 1979.